Bayesian Calibration using Gaussian Surrogate Model of the Likelihood Function: Application to Train Suspensions Monitoring

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Problematic

- Development of a Bayesian calibration method for a system with **functional stochastic** input and output when the **likelihood function** is expensive to compute;
- Procedure relying on the construction of a **Gaussian surrogate model** (see [3]) to address computational costs;
- Surrogate modeling of the likelihood function itself rather than the functional system output.

Classical Bayesian calibration

- Equation of the system associating output $\mathbf{Y}$ to parameters $\mathbf{W}$:

$$\mathbf{Y} = \mathbf{H}(\mathbf{W})$$

- Objective: Update the distribution of $\mathbf{W}$ from a measurement $\mathbf{y}^{\text{mes}}$ of $\mathbf{Y}$ using Bayes law:

$$p_{\mathbf{W}}^{\text{post}}(\mathbf{w}) = p_{\mathbf{W}\,|\,\mathbf{Y}}(\mathbf{w}\,|\,\mathbf{y}^{\text{mes}}) \propto p_{\mathbf{Y}\,|\,\mathbf{W}}(\mathbf{y}^{\text{mes}}\,|\,\mathbf{w}) \cdot p_{\mathbf{W}}^{\text{prior}}(\mathbf{w})$$

↳ Likelihood $\mathcal{L}(\mathbf{w})$

- Distribution $p_{\mathbf{W}}^{\text{post}}$ estimated with a **MCMC algorithm** (see [1]).

Industrial case: Train suspensions monitoring



Goal: Determine the state of the suspensions from joint measurements of the **track geometric irregularities** (see [2]) and of the **train dynamic response** (using embedded accelerometers).

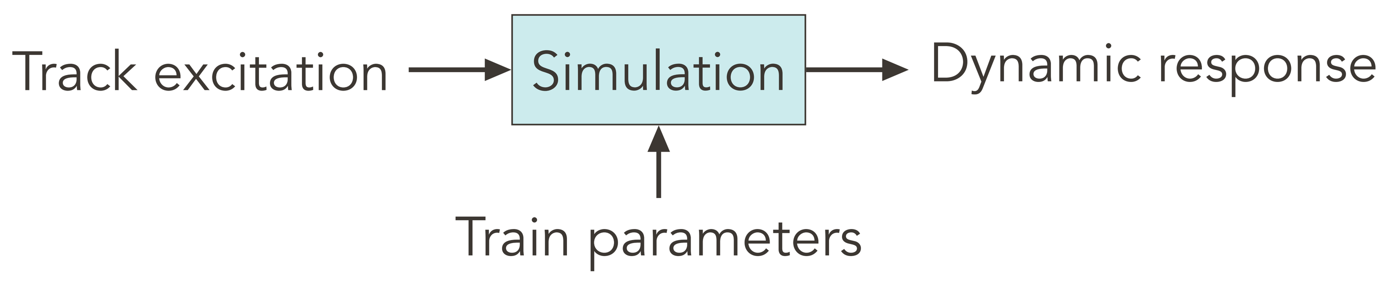

Figure 1: Diagram of the train dynamics system

Specificities of the studied case:

- Simulation-based model of the physical system;
- Simultaneous calibration of multiple parameters;
- Calibration with joint input-output measurements;
- Large quantity of available data.

Gaussian surrogate model

Likelihood function $\mathcal{L}$ expensive to compute:

→ Approximation by a Gaussian surrogate model $L(.;\Theta)$ of the log-likelihood;

→ Straightforward solution: use the predictor provided by the mean function $E_\Theta\{L(.;\Theta)\}$.

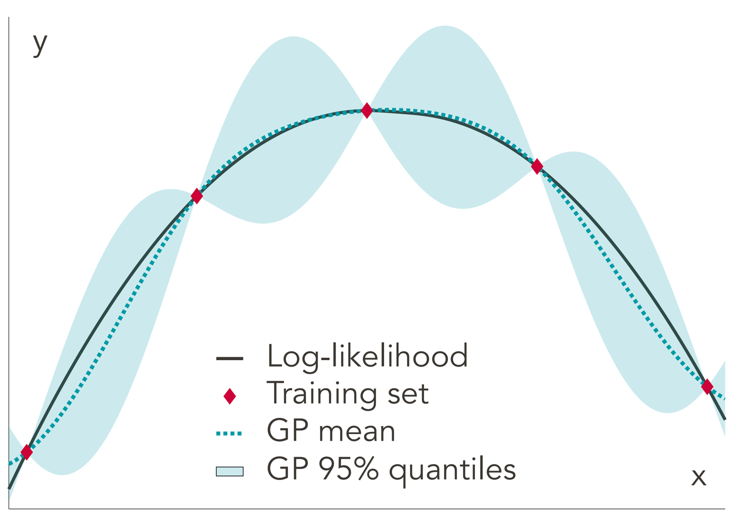

Figure 2: Gaussian surrogate model of the log-likelihood

Trajectory approximation

Computation of the Gaussian process trajectories ?

→ Approximation of a trajectory by further conditioning the Gaussian surrogate model on a set $\mathcal{W}$:

$$\widetilde{L}(.;\theta) : \mathbf{w} \mapsto = E_\Theta\left\{L(\mathbf{w};\Theta)\,|\,L(\mathcal{W};\Theta) = L(\mathcal{W};\theta)\right\}$$

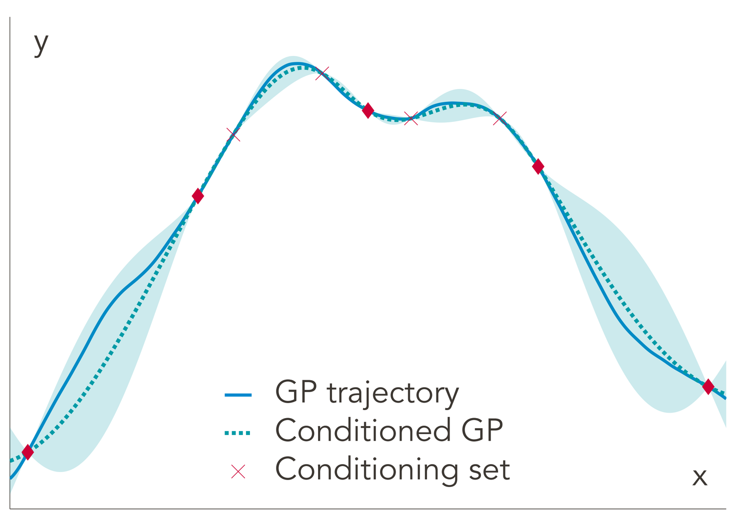

Figure 4: Trajectory approximation using the surrogate model

Surrogate model uncertainty

Influence of the surrogate model uncertainty on the estimated calibration accuracy ?

$$p_{\mathbf{W}\,|\,\mathbf{Y}} = E_\Theta\{p_{\mathbf{W}\,|\,\mathbf{Y},\Theta}\}$$

→ Monte Carlo sampling of trajectories of the surrogate model;

→ MCMC on the trajectories.

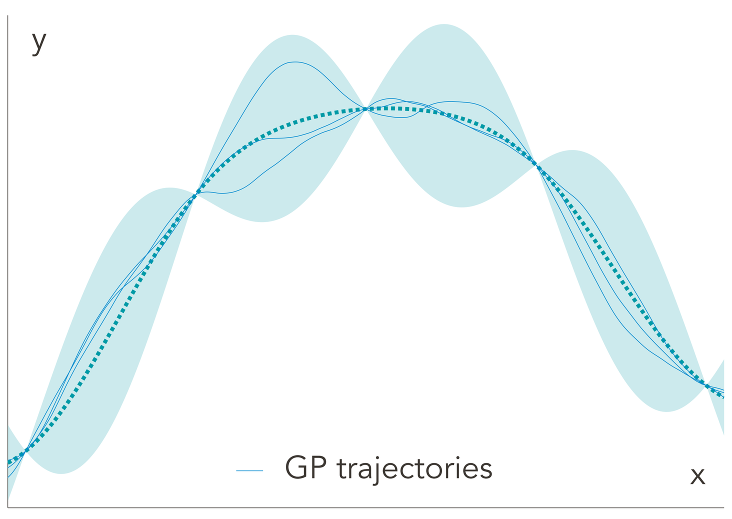

Figure 3: Example of trajectories of the Gaussian surrogate model

Choice of the conditioning set $\mathcal{W}$

Purpose: reduce as much as possible the variance of the surrogate model around the likelihood maximum.

→ Conditioning set $\mathcal{W}$ space-filling in the following set

$$\{\mathbf{w}\,|\,P(L(\mathbf{w};\Theta) > L(\mathbf{w}^{\max};\Theta)) \geq \rho\}$$

with $\mathbf{w}^{\max} = \arg\max_{\mathbf{w}} E_\Theta\{L(\mathbf{w};\Theta)\}$.

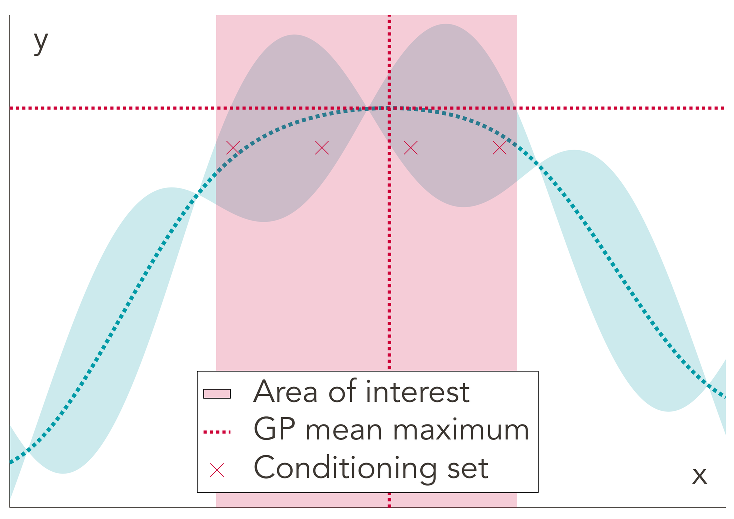

Figure 5: Definition of the conditioning set

Results on the railway case

- Validation of the proposed method on a **numerical experiment**;
- Visible evolution of the parameters from the nominal values when using actual measurements (Fig. 6);
- Significant influence of the trajectories method on the size of confidence intervals (Fig. 7).

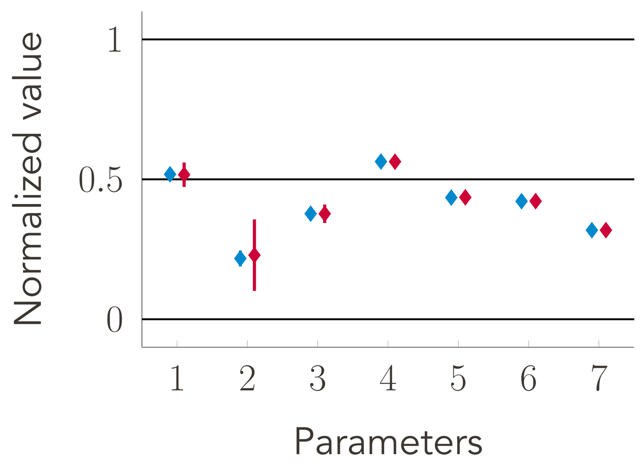

Figure 6: Calibration results: mean and 95% confidence intervals

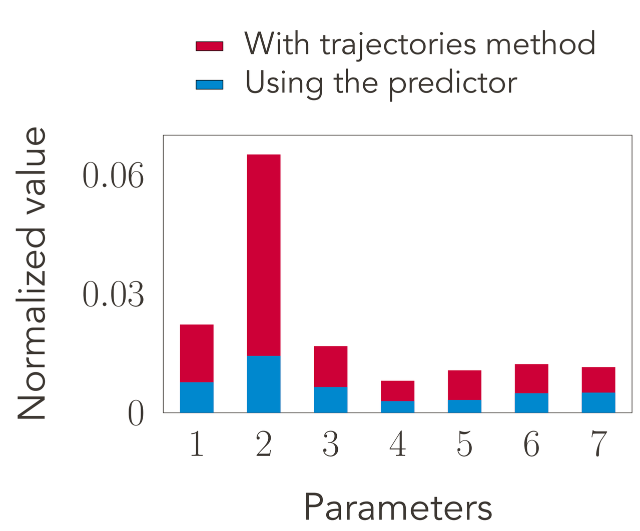

Figure 7: Standard deviation

References

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