Reliability-based sensitivity analysis under distribution parameter uncertainty – Application to aerospace systems

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Context & Motivations

- Evaluate the performances of complex aerospace systems
- Models, codes, variables rely on multiple sources of information affected by uncertainties (aleatory and epistemic)

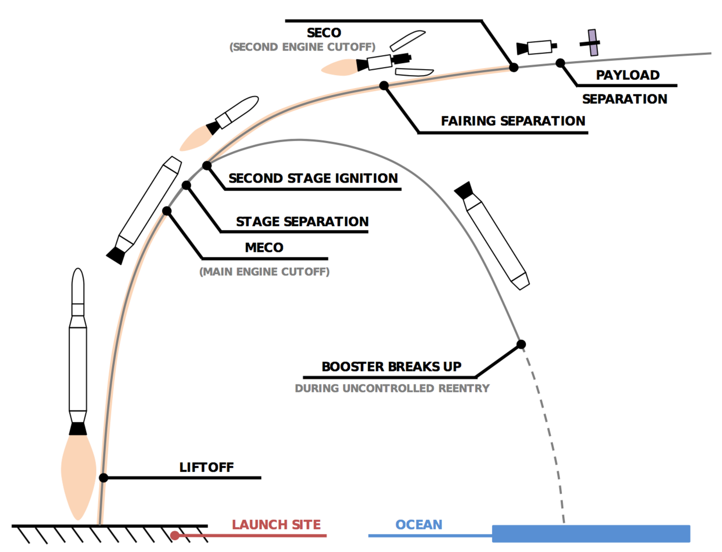


Figure: Launch vehicle fallback zone estimation.

Notations & Problem statement

- Expensive-to-evaluate deterministic & static computer code $\Longleftrightarrow$ **black box model**

$$\mathcal{M} : \mathbb{R}^d \to \mathbb{R},\ \mathbf{X} \mapsto Y = \mathcal{M}(\mathbf{X}) \quad (1)$$

- Limit-state function $\Longleftrightarrow$ **failure/safety of the system**

$$g : \mathbb{R}^d \to \mathbb{R},\ \mathbf{X} \mapsto g(\mathbf{X}) = y_{\text{th}} - Y \quad (2)$$

- Hierarchical model in input:
  - (level 1 - stochastic) $\mathbf{X} \sim f_{\mathbf{X}|\boldsymbol{\Theta}}(\mathbf{x}|\boldsymbol{\theta}) : \mathcal{D}_{\mathbf{X}} \subseteq \mathbb{R}^d \to \mathbb{R}_+$
  - (level 2 - stochastic) $\boldsymbol{\Theta} \sim f_{\boldsymbol{\Theta}|\boldsymbol{\xi}}(\boldsymbol{\theta}|\boldsymbol{\xi}) : \mathcal{D}_{\boldsymbol{\Theta}} \subseteq \mathbb{R}^k \to \mathbb{R}_+$
  - (level 3 - deterministic) $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_q)^\top \in \mathcal{D}_{\boldsymbol{\xi}} \subseteq \mathbb{R}^q$

Issues

How to deal with this **bi-level uncertainty**?

1. Choose an a priori model for the uncertainty affecting $\boldsymbol{\Theta}$ (**modeling**)
2. Propagate the bi-level uncertainty in the failure probability estimation (**propagation**)
3. Link the variability of the failure probability to the input uncertainty (**sensitivity**)

Uncertainty propagation via the Augmented Reliability Approach

- Conditional failure probability:

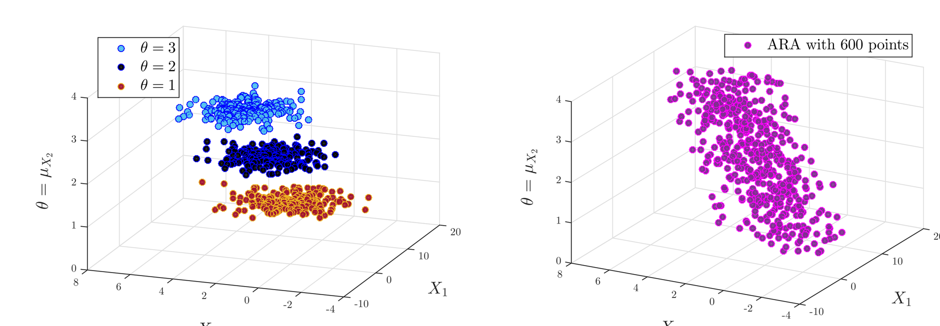
$$P_{\text{f}}(\boldsymbol{\theta}) = \mathbb{P}\left[g(\mathbf{X}) \leq 0 \mid \boldsymbol{\Theta} = \boldsymbol{\theta}\right] = \int_{\mathcal{D}_{\mathbf{X}}} \mathbb{1}_{\mathcal{F}_{\mathbf{x}}}(\mathbf{x}) f_{\mathbf{X}|\boldsymbol{\Theta}}(\mathbf{x}|\boldsymbol{\theta}) \mathrm{d}\mathbf{x} \quad (3a)$$

- Predictive failure probability (PFP) [1]:

$$\widetilde{P}_{\text{f}}(\boldsymbol{\xi}) = \int_{\mathcal{D}_{\boldsymbol{\Theta}}} \int_{\mathcal{D}_{\mathbf{X}}} \mathbb{1}_{\mathcal{F}_{\mathbf{x}}}(\mathbf{x}) f_{\mathbf{X}|\boldsymbol{\Theta}}(\mathbf{x}|\boldsymbol{\theta}) f_{\boldsymbol{\Theta}|\boldsymbol{\xi}}(\boldsymbol{\theta}|\boldsymbol{\xi}) \mathrm{d}\mathbf{x}\mathrm{d}\boldsymbol{\theta} \quad (4a)$$

$$= \int_{\mathcal{D}_{\mathbf{Z}}} \mathbb{1}_{\mathcal{F}_{\mathbf{z}}}(\mathbf{z}) f_{\mathbf{Z}|\boldsymbol{\xi}}(\mathbf{z}|\boldsymbol{\xi}) \mathrm{d}\mathbf{z} \quad (4b)$$

- Nested (NRA) vs. **Augmented Reliability Approach** (ARA)



(a) NRA (3×200 samples) (b) ARA (600 samples)

Figure: NRA vs. ARA for $X_1 \sim \mathcal{N}(7, 5/\sqrt{3})$, $X_2 \sim \mathcal{N}(\Theta, 2/\sqrt{3})$ and $\Theta \sim \mathcal{N}(2, 1.5)$.

↪ V. Chabridon et al. "Evaluation of failure probability under parameter epistemic uncertainty: application to aerospace system reliability assessement". *Aerospace Science and Technology* 69 (2017) 526-537.

↪ V. Chabridon et al. "Some Bayesian insights for statistical tolerance analysis". 23^{ième} *Congrès Français de Mécanique (CFM 2017)*, Lille.

Local reliability-based sensitivity analysis

- Derivations of local sensitivity estimators of the PFP w.r.t. deterministic hyper-parameters:

1. Unbounded distribution for $\boldsymbol{\Theta}$:

$$S_j \overset{\text{def}}{=} \frac{\partial \widetilde{P}_{\text{f}}(\boldsymbol{\xi})}{\partial \xi_j} \underset{\text{MC}}{\approx} \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}_{\mathcal{F}_{\mathbf{z}}}(\mathbf{Z}^{(i)}) \kappa_j(\boldsymbol{\Theta}^{(i)}, \boldsymbol{\xi}) \quad (5)$$

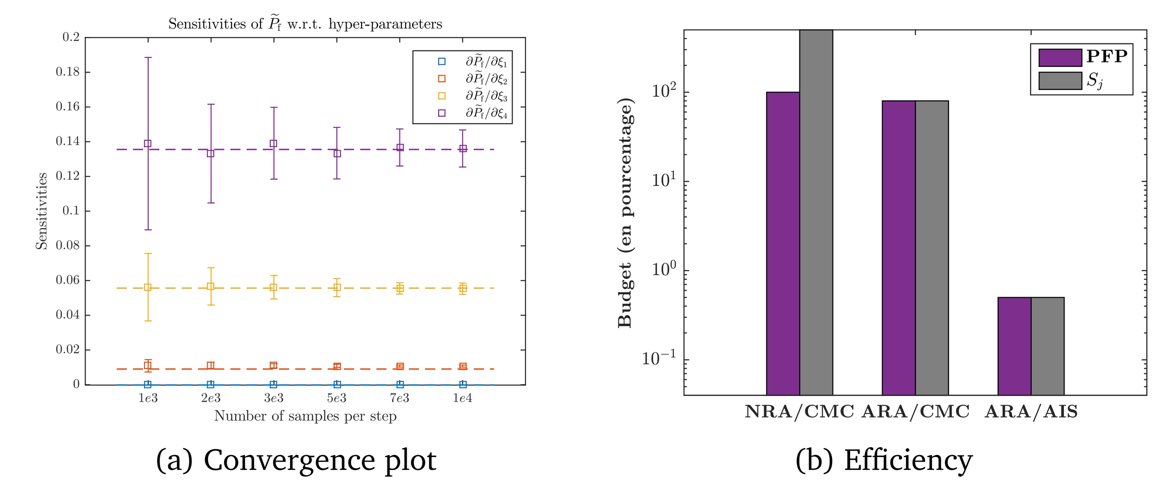
2. Bounded distribution [2] for $\boldsymbol{\Theta}$:

$$S_j \propto f_{X_j|\Theta_j}(x_j|\theta_j = \text{u/l-bound}) \left(\widetilde{P}_{\text{f}} - P_{\text{f,aux}}^{\theta_j = \text{u/l-bound}} \right) \quad (6)$$

- Estimation strategy: augmented Nonparametric Adaptive Importance Sampling (NAIS)

↪ V. Chabridon et al. "Reliability-based sensitivity analysis of aerospace systems under distribution parameter uncertainty using an augmented approach". *ICOSSAR 2017*, Vienna.

↪ V. Chabridon et al. "Reliability-based sensitivity estimators of rare event probability under distribution parameter uncertainty". (Under Review) *Reliability Engineering & System Safety*.



(a) Convergence plot (b) Efficiency

Figure: Estimation results for ARA/NAIS.

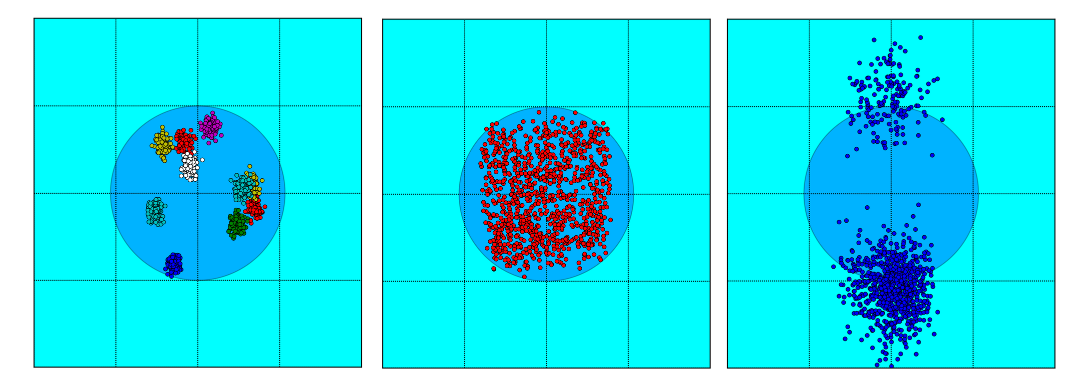


Figure: Illustration of the simulation of first launcher stage impact points into the ocean (10^3 samples for each method): NRA/CMC – ARA/CMC – ARA/NAIS

↪ V. Chabridon et al. "Nonparametric adaptive importance sampling strategy for reliability assessment and sensitivity analysis under distribution parameter uncertainty - Application to launch vehicle fallback zone estimation". 10^{ième} *Journées Fiabilité des Matériaux et des Structures (JFMS 2018)*, Bordeaux.

↪ P. Derennes, V. Chabridon et al. "Advanced numerical strategies for sensitivity analysis and reliability assessment of a launcher stage fall-back zone estimation simulation code". (Under Review) G. Fasano, J. Pintér (Eds.) *Optimization in Space Engineering*, Springer International Publishing.

Global reliability-based sensitivity analysis

- Starting from the definition of global reliability-oriented Sobol' indices [3]:

1. First index:

$$s_i = \frac{\mathbb{V}\left[\mathbb{E}\left[\mathbb{1}_{\mathcal{F}_{\mathbf{x}}}|x_i\right]\right]}{\mathbb{V}\left[\mathbb{1}_{\mathcal{F}_{\mathbf{x}}}\right]} \quad (7)$$

2. Total index:

$$t_i = 1 - \frac{\mathbb{V}\left[\mathbb{E}\left[\mathbb{1}_{\mathcal{F}_{\mathbf{x}}}|\mathbf{x}^{\sim i}\right]\right]}{\mathbb{V}\left[\mathbb{1}_{\mathcal{F}_{\mathbf{x}}}\right]} \quad (8)$$

- The second uncertainty level can be considered using a transformation of random variables [4]
- Efficient estimation strategy using post-treatment of Subset Simulations [5]
- Main goal: get separate global sensitivity indices for both contributions (variability and parameter uncertainty) [6]

Conclusions

- $\widetilde{P}_{\text{f}}(\boldsymbol{\xi})$ can be a relevant measure of safety for design purposes (but may not be sufficient for risk management)
- Robustness of this measure w.r.t. some priori choices is investigated in terms of sensitivity analysis
- Local and global sensitivities can be obtained by efficient numerical sampling compared to CMC

Perspectives

- To use metamodels to speed up the calculations or to get the sensitivity measures as by-products of the metamodeling step

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