Quantile prediction of a random field extending the gaussian setting

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Introduction

Conditional quantiles of a random field may be usefull (Probability of Improvement, confidence bounds for kriging prediction).

Quite simple in the gaussian case.

What about elliptical random fields?
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Elliptical distributions

**Definition ([Cambanis et al., 1981])**

Let $X$ be a random vector of dimension $d$. $X$ is said elliptical iff there exists a unique $\mu \in \mathbb{R}^d$, a positive definite matrix $\Sigma \in \mathbb{R}^{d \times d}$, and a non-negative random variable $R$ such that:

$$X \stackrel{d}{=} \mu + R \Lambda U^{(d)}$$

with $\Lambda \Lambda^T = \Sigma$, $U^{(d)}$ is a uniform distribution on the unit sphere of dimension $d$, independent of $R$. Furthermore, $X$ is consistent if $R \stackrel{d}{=} \sqrt{\chi^2_d} \xi$, where $\xi$ is a non-negative random variable independent of $\chi^2_d$ and $d$. [Kano, 1994]
Elliptical distributions

Theorem (Elliptical density)

If $X \sim \mathcal{E}_d(\mu, \Sigma, R)$, then:

$$f_X(x) = \frac{c_d}{|\det(\Sigma)|^{\frac{1}{2}}} g_d \left( ((x - \mu)\Sigma^{-1}(x - \mu)) \right)$$

where $c_d g_d(t) = \frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{2\pi^2}} \sqrt{t^{-(d-1)}} f_R(\sqrt{t})$, and $f_R(t)$ is the p.d.f of $R$.

- $g_d$ is called the generator of $X$. 
Examples

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Constant $c_d$</th>
<th>Generator $g_d(t)$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>$\frac{1}{(2\pi)^{d/2}}$</td>
<td>$\exp\left(-\frac{t}{2}\right)$</td>
<td>1</td>
</tr>
<tr>
<td>Student, $\nu &gt; 0$</td>
<td>$\frac{\Gamma\left(\frac{d+\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{(\nu \pi)^{d/2}}$</td>
<td>$\left(1 + \frac{t}{\nu}\right)^{-\frac{d+\nu}{2}}$</td>
<td>$\frac{\nu}{\sqrt{\chi^2_{\nu}}}$</td>
</tr>
<tr>
<td>Gaussian Mixture</td>
<td>$\frac{1}{(2\pi)^{d/2}}$</td>
<td>$\sum_{k=1}^{n} \pi_k \theta_k^{-d} \exp\left(-\frac{1}{2\theta_k^2} t\right)$</td>
<td>$\sum_{k=1}^{n} \pi_k \delta \theta_k$</td>
</tr>
<tr>
<td>Slash, $a &gt; 0$</td>
<td>$2^{\frac{d}{2}-1} a \Gamma\left(\frac{d+a}{2}\right)$</td>
<td>$\frac{\chi_{d+a}(t)}{t^{d+a}}$</td>
<td>Pareto(1, a)</td>
</tr>
</tbody>
</table>
Definition

A random field \( \{X(t)\}_{t \in T} \) is \( \xi \)-elliptical if for all \( N \in \mathbb{N} \) and all \( t_1, \ldots, t_N \in T \), the vector \((X(t_1), \ldots, X(t_N))\) is \((\xi, N)\)-elliptical.
Elliptical random fields

Gaussian random field

Contaminated Gaussian random field

Student random field

Slash random field
Let \( \{X(t)\}_{t \in T} \) be a \( \xi \)-elliptical random field. The aim is to provide \( q_{\alpha}(Y|X = x) \) where \( Y = X(t), t \in T \) and \( X = (X(t_1), \ldots, X(t_N)), t_1, \ldots, t_N \in T \). Notice that \( \alpha = 1/2 \) leads to classical kriging.

Theoretical conditional quantiles:

\[
q_{\alpha}(Y|X = x) = \mu_{Y|X} + \sigma_{Y|X} \Phi_{R^*}^{-1}(\alpha)
\]

with

\[
\begin{align*}
\mu_{Y|X} &= \mu_Y + \Sigma_{YX} \Sigma_X^{-1}(x - \mu_X) \\
\sigma^2_{Y|X} &= \Sigma_Y - \Sigma_{YX} \Sigma_X^{-1} \Sigma_{XY}
\end{align*}
\]

**Problem**: Estimation of \( \Phi_{R^*}^{-1}(\alpha) \).
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A first idea is to use Quantile Regression:

\[ q^{QR}_\alpha(Y|X = x) = \beta^*^\top X + \beta_0^* , \]

where

\[ (\beta^*, \beta_0^*) = \arg\min_{\beta \in \mathbb{R}^N, \beta_0 \in \mathbb{R}} \mathbb{E} \left[ S_\alpha \left( Y - \beta^\top X - \beta_0 \right) \right] \]

and

\[ S_\alpha(s) = (\alpha - 1)s + \max \{ s, 0 \} . \]
**Theorem ([Maume-Deschamps et al., 2017a])**

The quantile regression vector \((\beta^*, \beta_0^*)\) of \(Y|(X = x)\), satisfying the previous problem, is given by

\[
\beta^* = \Sigma_X^{-1}\Sigma_{XY}, \quad \beta_0^* = \mu_Y - \Sigma_{YX}\Sigma_X^{-1}\mu_X + \sigma_{Y|X}\Phi_R^{-1}(\alpha).
\]

The quantile regression predictor with level \(\alpha \in [0, 1]\) is given by

\[
q_{\alpha}^{QR}(Y|X = x) = \mu_{Y|X} + \sigma_{Y|X}\Phi_R^{-1}(\alpha).
\]

Furthermore, the distribution of \(q_{\alpha}^{QR}(Y|X)\) is

\[
q_{\alpha}^{QR}(Y|X) \sim \mathcal{E}_1 \left\{ \mu_Y + \sigma_{Y|X}\Phi_R^{-1}(\alpha), \Sigma_{YX}\Sigma_X^{-1}\Sigma_{XY}, \xi \right\}.
\]
- Inefficient linear model.
- $\Phi_{R}^{-1}(\alpha)$ instead of $\Phi_{R^*}^{-1}(\alpha)$.
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Some asymptotic relationships

Theorem ([Maume-Deschamps et al., 2017a])

Under some assumptions, there exist $0 < \ell < +\infty$ and $\eta \in \mathbb{R}$ such that:

$$
\left[ \Phi^{-1}_R \left( 1 - \frac{1}{1-\alpha + 2(1-\ell)} \right) \right]^{\frac{1}{\eta}} \sim_{\alpha \to 1} \Phi^{-1}_{R^*}(\alpha)
$$

We thus define the following approximation for $q_\alpha(Y|X = x)$:

$$
q^E_\alpha(Y|X = x) = \mu_{Y|x} + \sigma_{Y|x} \left[ \Phi^{-1}_R \left( 1 - \frac{1}{\frac{\ell}{1-\alpha}+2(1-\ell)} \right) \right]^{\frac{1}{\eta}}
$$
### Proposition ([Maume-Deschamps et al., 2017a])

The Gaussian, Student, Gaussian Mixture, and Slash distributions satisfy the previous assumptions, with coefficients \(\eta\) and \(\ell\) given in the table below.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>(\eta)</th>
<th>(\ell)</th>
</tr>
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<tbody>
<tr>
<td>Gaussian</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Student, (\nu &gt; 0)</td>
<td>(\frac{N}{\nu} + 1)</td>
<td>[\frac{\Gamma\left(\frac{\nu+N+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+N}{2}\right)\Gamma\left(\frac{\nu+1}{2}\right)}\left(1 + \frac{q_1}{\nu}\right)\frac{N+\nu}{\nu} - \frac{N+1}{\nu+N}]</td>
</tr>
<tr>
<td>Gaussian Mixture</td>
<td>1</td>
<td>[\min(\theta_1^{-1}, \ldots, \theta_n^{-1})^N \exp\left(-\frac{\min(\theta_1^{-1}, \ldots, \theta_n^{-1})^2}{2} q_1\right) ]</td>
</tr>
<tr>
<td>Slash, (a &gt; 0)</td>
<td>(\frac{N}{a} + 1)</td>
<td>[\frac{\Gamma\left(\frac{N+1+a}{2}\right)q_1^{\frac{N+a}{2}}}{\Gamma\left(\frac{N+a}{2}\right)(N+a)\chi_{N+a}^2(q_1)2^{\frac{a}{2}-1}\Gamma\left(\frac{1+a}{2}\right)}]</td>
</tr>
</tbody>
</table>
Estimation ([Usseglio-Carleve, 2017])

- Let \((X_1, Y_1), \ldots, (X_n, Y_n)\) be a random sample independent and identically distributed from an \((\xi, N + 1)\)-elliptical vector with \((\mu, \Sigma)\) known.

**Assumption**

*We assume that there exist a function \(A\) such that \(A(t) \to 0\) as \(t \to +\infty\), and*

\[
\lim_{t \to +\infty} \frac{\Phi^{-1}_R\left(1 - \frac{t}{\omega t}\right) - \omega^\gamma}{\Phi^{-1}_R\left(1 - \frac{1}{t}\right)} A(t) = \omega^\gamma \frac{\omega^\rho - 1}{\rho}
\]

*where \(\gamma > 0\) and \(\rho \leq 0\).*
Parameters estimation

Proposition

Under the previous assumption, parameters $\eta$ and $\ell$ exist, and are expressed:

$$
\begin{aligned}
\eta &= 1 + \gamma N \\
\ell &= \frac{1}{\Gamma \left( \frac{N+\gamma^{-1}+1}{2} \right)} \frac{\Gamma \left( \frac{\gamma^{-1}+1}{2} \right)}{\Gamma \left( \frac{\gamma^{-1}}{2} \right)} \left( \frac{\gamma^{-1} \pi - \frac{N}{2}}{(N+\gamma^{-1})c_{NG_N}(m(x))} \right).
\end{aligned}
$$

where $m(x) = (x - \mu_X)^T \Sigma_X^{-1} (x - \mu_X)$.
Definition

We define the two following estimators:

\[
\hat{\eta}_{kn} = \begin{cases} 
\hat{\eta}_{kn} = & N \hat{\gamma}_{kn} + 1 \\
\hat{\ell}_{kn}, h_n = & \frac{\Gamma\left(\frac{N + \hat{\gamma}_{kn}^{-1} + 1}{2}\right)}{\Gamma\left(\frac{\hat{\gamma}_{kn}^{-1} + 1}{2}\right)} \left(N + \hat{\gamma}_{kn}^{-1}\right) \frac{m(x)^{\frac{1}{2}} - \frac{N}{2} \Gamma\left(\frac{N}{2}\right)}{\pi n h_n} \sum_{i=1}^{n} K \left(\frac{m(x) - (X_i - \mu_X)^T \Sigma^{-1} (X_i - \mu_X)}{h_n}\right)
\end{cases}
\]

where \( \hat{\gamma}_{kn} = \frac{1}{k_n} \sum_{i=1}^{k_n} \ln \left(\frac{W[i]}{W[k_n+1]}\right), \) \( k_n = o(n), \) \( h_n = o(1), \) \( k_n \to +\infty, \)

\( nh_n \to +\infty \) as \( n \to +\infty \) and \( W \) is the first (or indifferently any) component of the vector \( \Lambda_X^{-1}(X - \mu_X). \)
Parameters estimation

Proposition

Under our assumption, and if \( \sqrt{k_n} A \left( \frac{n}{k_n} \right) \to 0 \) as \( n \to +\infty \), then the following asymptotic relationships hold:

- If \( nh_n/k_n \to 0 \) as \( n \to +\infty \), then

\[
\sqrt{nh_n} \left( \hat{\ell}_{k_n, h_n} - \ell \right) \sim_{n \to +\infty} N \left( \left( 0, \left( V_2 \ 0 \right) \right) \right)
\]

where \( V_2 = \)

\[
\frac{\Gamma \left( \frac{N}{2} \right)}{m(x)^{\frac{N}{2}-1} \pi^{\frac{N}{2}}} c_N g_N(m(x)) \int K(u)^2 du \left[ \frac{\Gamma \left( \frac{N+\gamma^{-1}+1}{2} \right)}{\Gamma \left( \frac{\gamma^{-1}+1}{2} \right)} (N + \gamma^{-1})^{-1} \gamma^{-1} \pi - \frac{N}{2} \right]^2
\]
Under our assumption, and if $\sqrt{k_n}A\left(\frac{n}{k_n}\right) \to 0$ as $n \to +\infty$, then the following asymptotic relationships hold:

- If $nh_n/k_n \to +\infty$, then

$$
\sqrt{k_n}\left(\ell_{k_n,h_n} - \ell\right) \xrightarrow{n \to +\infty} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} V_1 & -N\gamma\sqrt{V_1} \\ -N\gamma\sqrt{V_1} & N^2\gamma^2 \end{pmatrix}\right)
$$

where $V_1 =$

$$
\frac{\pi^{-N\gamma^2}}{c_N g_N(m(x))^2} \frac{\Gamma\left(\frac{N+\gamma^{-1}+1}{2}\right)^2}{\Gamma\left(\frac{\gamma^{-1}+1}{2}\right)^2} \frac{\psi\left(\frac{\gamma^{-1}+1}{2}\right) - \psi\left(\frac{N+\gamma^{-1}+1}{2}\right)}{2\gamma^2 \left(N\gamma + 1\right)} - \frac{N}{\left(N\gamma + 1\right)^2}
$$
Proposition

Under our assumption, and if \( \sqrt{k_n} A \left( \frac{n}{k_n} \right) \to 0 \) as \( n \to +\infty \), then the following asymptotic relationships hold:

- If \( nh_n / k_n \xrightarrow[n \to +\infty]{} c \in \mathbb{R}^* \), then

\[
\sqrt{k_n} \begin{pmatrix} \hat{\ell}_{kn, h_n} - \ell \\ \hat{\eta}_{kn} - \eta \end{pmatrix} \xrightarrow[n \to +\infty]{} \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} V_1 + \frac{1}{c}V_2 & -N\gamma \sqrt{V_1} \\ -N\gamma \sqrt{V_1} & N^2\gamma^2 \end{pmatrix} \right)
\]
We consider \((\alpha_n)_n\) such that \(\alpha_n \to 1\) as \(n \to +\infty\), and distinguish two cases:

- **Intermediate quantiles.** We suppose \(n(1 - \alpha_n) \to +\infty\). It entails that the estimation of the \(\alpha_n\)-quantile leads to an interpolation of sample results.

- **High quantiles.** We suppose \(n(1 - \alpha_n) \to 0\), i.e. we need to extrapolate sample results to areas where no data are observed.
Quantile estimation

Definition

We define the two following estimators, respectively for intermediate and high quantiles:

\[
\begin{align*}
\hat{q}_{\alpha_n}^E (Y|X = x) &= \mu_{Y|x} + \sigma_{Y|x} \left( W[k_{n+1}] \right)^{\frac{1}{\hat{\eta}_{kn}}} \\
\hat{q}_{\alpha_n}^E (Y|X = x) &= \mu_{Y|x} + \sigma_{Y|x} \left[ W[k_{n+1}] \left( \frac{k_n}{n} \left( 2 + \hat{\ell}_{kn,h_n} \left( \frac{1}{1-\alpha_n} - 2 \right) \right) \right)^{\frac{1}{\hat{\eta}_{kn}}} \right]
\end{align*}
\]

where \( \tilde{k}_n = \frac{n}{2 + \hat{\ell}_{kn,h_n} \left( \frac{1}{1-\alpha_n} - 2 \right) \right) \) and \( W \) is the first (or indifferently any) component of the vector \( \Lambda_X^{-1} (X - \mu X) \).
Consider that our assumption holds and assume that:

1. \( k_n = o(nh_n) \) and \( \sqrt{k_n}A\left(\frac{n}{k_n}\right) \to 0 \) as \( n \to +\infty \).

2. \( n(1 - \alpha_n) \to +\infty \), \( \ln(1 - \alpha_n) = o(\sqrt{k_n}) \) and \( \frac{\sqrt{k_n}}{\ln(1 - \alpha_n)} = o\left(\sqrt{n(1 - \alpha_n)}\right) \) as \( n \to +\infty \).

Then

\[
\frac{\sqrt{k_n}}{\ln(1 - \alpha_n)} \left( \frac{\hat{q}_{\alpha_n}^E(Y|X = x)}{q_{\alpha_n}^E(Y|X = x)} - 1 \right) \xrightarrow{n \to +\infty} \mathcal{N}\left(0, \frac{N^2\gamma^4}{\gamma N + 1)^4}\right).
\]

Therefore:

\[
\frac{\hat{q}_{\alpha_n}^E(Y|X = x)}{q_{\alpha_n}(Y|X = x)} \xrightarrow{P} 1 \text{ as } n \to +\infty.
\]
Theorem

We denote $p_n = \left(2 + \ell ((1 - \alpha_n)^{-1} - 2)\right)^{-1}$, $\tilde{p}_n = \left(2 + \ell \hat{k}_n, h_n ((1 - \alpha_n)^{-1} - 2)\right)^{-1}$.

Consider that our assumption holds and assume that:

- $k_n = o(nh_n)$ and $\sqrt{k_n}A \left(\frac{n}{k_n}\right) \to 0$ as $n \to +\infty$.
- $n(1 - \alpha_n) \to 0$, $\ln (n(1 - \alpha_n)) = o(\sqrt{k_n})$ and $\frac{\ln (1 - \alpha_n)}{\ln \left(\frac{n}{k_n} (1 - \alpha_n)\right)} \to \theta \in [0, +\infty[.

Then

$$\frac{\sqrt{k_n}}{\ln \left(\frac{n}{k_n} (1 - \alpha_n)\right)} \left(\frac{\hat{q}_{\alpha_n}^E (Y|X = x)}{q_{\alpha_n}^E (Y|X = x)} - 1\right) \overset{\mathcal{N}}{\to} N \left(0, \left(\frac{\gamma}{\gamma N + 1} - \theta \frac{\gamma^2 N}{(\gamma N + 1)^2}\right)^2\right)$$

Therefore:

$$\frac{\hat{q}_{\alpha_n}^E (Y|X = x)}{q_{\alpha_n} (Y|X = x)} \overset{\mathbb{P}}{\to} 1 \text{ as } n \to +\infty.$$
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Definition ([Chen, 1996])

Let $Z$ be a real random variable. The $L_p$-quantiles of $Z$ with level $\alpha \in ]0, 1[$ and $p > 0$, denoted $q_{p,\alpha}(Z)$, is solution of the minimization problem:

$$q_{p,\alpha}(Z) = \arg \min_{z \in \mathbb{R}} \mathbb{E} \left[ (1 - \alpha) (z - Z)_+^p + \alpha (Z - z)_+^p \right]$$

where $Z_+ = Z \mathbb{1}_{\{z > 0\}}$. 

**$L_p$-quantiles**

**Perspectives**

Simulation study
According to [Koenker and Bassett, 1978], the case $p = 1$ leads to the quantile $q_{1,\alpha}(Z) = F_Z^{-1}(\alpha)$.

The case $p = 2$ is called expectile, which knows a growing interest: [Taylor, 2008], [Cai and Weng, 2016], [Maume-Deschamps et al., 2017b].

Other cases are difficult to deal with: [Bernardi et al., 2017].
**Theorem** ([Daouia et al., 2017])

*If a random variable \( Z \) fills our assumption, then*

\[
\frac{q_{p,\alpha}(Z)}{q_\alpha(Z)} \xrightarrow{\alpha \to 1} \left[ \frac{\gamma}{\beta(p, \gamma^{-1} - p + 1)} \right]^{-\gamma} = f_L(\gamma, p)
\]

*where \( \beta \) is the beta function.*

**Lemma**

*Under our assumption, the conditional distribution \( Y|X = x \) is attracted to a maximum domain of Pareto-type distribution with tail index \((\gamma^{-1} + N)^{-1}\).*
Conditional extreme $L_p$-quantiles estimation

**Definition**

Let $(\alpha_n)_{n \in \mathbb{N}}$ be a sequence such that $\alpha_n \to 1$ as $n \to +\infty$. We define:

\[
\begin{align*}
\hat{q}_{p,\alpha_n}(Y \mid X = x) &= \mu_Y \mid x + \sigma_Y \mid x \left( W[2\tilde{p}_n + 1] \right)^{\frac{1}{\hat{n}_kn}} f_L \left( \hat{\gamma}_{kn}^{-1} + N \right)^{-1}, p \\
\hat{q}_{p,\alpha_n}(Y \mid X = x) &= \mu_Y \mid x + \sigma_Y \mid x \left[ W[kn + 1] \left( \frac{kn}{\tilde{p}_n} \left( 2 + \hat{\ell}_{kn}, h_n \left( \frac{1}{1 - \alpha_n} - 2 \right) \right) \right] \right)^{\frac{1}{\hat{n}_kn}} f_L \left( \hat{\gamma}_{kn}^{-1} + N \right)^{-1}, p \\
\end{align*}
\]
**Haezendonck-Goovaerts risk measures**

**Definition ([Haezendonck and Goovaerts, 1982])**

Let $Z$ be a real random variable, and $\varphi$ a non negative and convex function with $\varphi(0) = 0$, $\varphi(1) = 1$ and $\varphi(+\infty) = +\infty$. The Haezendonck-Goovaerts risk measure of $Z$ with level $\alpha \in ]0, 1[$ associated to $\varphi$, is given by the following:

$$H_\alpha(Z) = \inf_{z \in \mathbb{R}} \{z + H_\alpha(Z, z)\}$$

where $H_\alpha(Z, z)$ is the unique solution $h$ to the equation:

$$\mathbb{E} \left[ \varphi \left( \frac{(Z - z)_+}{h} \right) \right] = 1 - \alpha$$

$\varphi$ is called Young function.
The case $\varphi(t) = t$ leads to the Tail Value-at-Risk $\text{TVaR}_\alpha(Z)$.

**Proposition ([Tang and Yang, 2012])**

If $Z$ fills our assumption, and taking a Young function $\varphi(t) = t^p, p \geq 1$, then the following relationship holds:

$$
\frac{H_{p,\alpha}(Z)}{q_{\alpha}(Z)} \overset{\alpha \to 1}{\sim} \frac{\gamma^{-1} (\gamma^{-1} - p)^{p\gamma - 1}}{p^{\gamma(p-1)}} \beta \left( \gamma^{-1} - p, p \right)^{\gamma} = f_H(\gamma, p)
$$

Antoine Usseglio-Carleve

Quantile prediction of elliptical random fields
Conditional extreme H-G risk measures estimation

Definition

Let \((\alpha_n)_{n \in \mathbb{N}}\) be a sequence such that \(\alpha_n \to 1\) as \(n \to +\infty\). We define:

\[
\begin{align*}
\hat{H}_{p, \alpha_n}(Y|X = x) &= \mu_{Y|x} + \sqrt{\Sigma_{Y|X}} \left( W_{[n \tilde{p} + 1]} \right) \frac{1}{\hat{\eta}_{kn}} f_H \left( \left( \hat{\gamma}_{kn}^{-1} + N \right)^{-1}, p \right) \\
\hat{H}_{p, \alpha_n}(Y|X = x) &= \mu_{Y|x} + \sqrt{\Sigma_{Y|X}} \left[ W_{[kn + 1]} \left( \frac{kn}{n} \left( 2 + \hat{\ell}_{kn}, h_n \left( \frac{1}{1 - \alpha_n} - 2 \right) \right) \right) \hat{\gamma}_{kn} \right] \frac{1}{\hat{\eta}_{kn}} \times f_H \left( \left( \hat{\gamma}_{kn}^{-1} + N \right)^{-1}, p \right)
\end{align*}
\]
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Simulation study

- We apply our estimators to a sample of a centered Student process with $\nu = 1.5$ degrees of freedom, and compare with theoretical results.
- $\sigma(t) = e^{-|t|}$.
- We take the sequences $k_n = n^{0.6}$, $h_n = n^{-0.15}$ and $\alpha_n = 1 - n^{-1.1}$.
- The chosen kernel $K$ is the gaussian p.d.f.
- Theoretical result is $q_{\alpha}(Y|X = x) = \sqrt{\frac{\nu + m(x)}{\nu + N}} \Phi_{\nu + N}(\alpha)$. 
Simulation study

Student process

- Regression
- Theoretical
- Extreme
References


working paper.
Merci !