Quantile prediction of a random field extending the gaussian setting

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Introduction

- Conditional quantiles of a random field may be usefull (Probability of Improvement, confidence bounds for kriging prediction).
- Quite simple in the gaussian case.
- What about elliptical random fields ?

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3. Quantile regression
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Elliptical distributions

Definition ([Cambanis et al., 1981])

Let X be a random vector of dimension d. X is said elliptical iff there exists a unique $\mu \in \mathbb{R}^d$, a positive definite matrix $\Sigma \in \mathbb{R}^{d \times d}$, and a non-negative random variable R such that :

$$X \stackrel{d}{=} \mu + R \Lambda U^{(d)}$$

with $\Lambda \Lambda^T = \Sigma$, $U^{(d)}$ is a uniform distribution on the unit sphere of dimension d, independant of R. Furthermore, X is consistent if $R \stackrel{d}{=} \sqrt{\chi_d^2} \xi$, where ξ is a non-negative random variable independant of χ_d^2 and d. [Kano, 1994]

Elliptical distributions

Theorem (Elliptical density)

If $X \sim \mathcal{E}_d(\mu, \Sigma, R)$, *then :*

$$f_X(x) = \frac{c_d}{|\det(\Sigma)|^{\frac{1}{2}}} g_d\left((x-\mu)\Sigma^{-1}(x-\mu)\right)$$

where $c_d g_d(t) = \frac{\Gamma\left(\frac{d}{2}\right)}{2\pi^{\frac{d}{2}}} \sqrt{t}^{-(d-1)} f_R(\sqrt{t})$, *and* $f_R(t)$ *is the p.d.f of* R.

- g_d is called the generator of X.

Examples

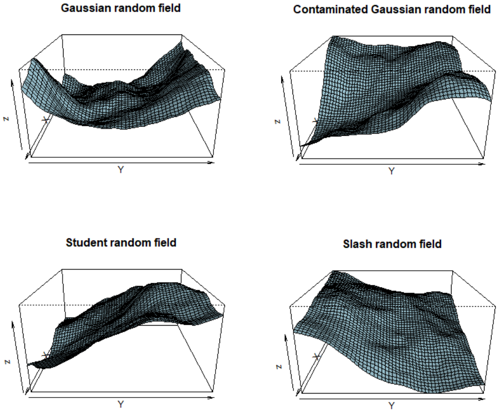
Distribution	Constant c_d	Generator $g_d(t)$	ξ
Gaussian	$\frac{1}{(2\pi)^{\frac{d}{2}}}$	$\exp(-\frac{t}{2})$	1
Student, $\nu > 0$	$\frac{\Gamma(\frac{d+\nu}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{(\nu\pi)^{\frac{d}{2}}}$	$\left(1 + \frac{t}{\nu}\right)^{-\frac{d+\nu}{2}}$	$\frac{\nu}{\sqrt{\chi^2_\nu}}$
Gaussian Mixture	$\frac{1}{(2\pi)^{\frac{d}{2}}}$	$\sum_{k=1}^{n} \pi_k \theta_k^{-d} \exp\left(-\frac{1}{2\theta_k^2} t\right)$	$\sum_{k=1}^{n} \pi_k \delta_{\theta_k}$
Slash, $a > 0$	$\frac{2^{\frac{a}{2}-1} a \Gamma\left(\frac{d+a}{2}\right)}{\pi^{\frac{d}{2}}}$	$\frac{\chi^2_{d+a}(t)}{t^{\frac{d+a}{2}}}$	$\text{Pareto}(1, a)$

Elliptical random fields

Definition

A random field $\{X(t)\}_{t\in T}$ is $\xi-$elliptical if for all $N \in \mathbb{N}$ and all $t_1, \ldots, t_N \in T$, the vector $(X(t_1), \ldots, X(t_N))$ is $(\xi, N)-$elliptical.

Elliptical random fields



Problem

Let $\{X(t)\}_{t\in T}$ be a $\xi-$elliptical random field. The aim is to provide $q_\alpha(Y|X=x)$ where $Y=X(t), t\in T$ and $X=(X(t_1),\ldots,X(t_N))\,, t_1,\ldots,t_N \in T$. Notice that $\alpha=1/2$ leads to classical kriging.

Theoretical conditional quantiles :

$$q_\alpha(Y|X=x)=\mu_{Y|x}+\sigma_{Y|X}\Phi_{R^*}^{-1}(\alpha)$$

with

$$\begin{cases} \mu_{Y|x} = & \mu_Y+\Sigma_{YX}\Sigma_X^{-1}(x-\mu_X) \\ \sigma_{Y|X}^2 = & \Sigma_Y-\Sigma_{YX}\Sigma_X^{-1}\Sigma_{XY} \end{cases}$$

<u>Problem</u> : Estimation of $\Phi_{R^*}^{-1}(\alpha)$.

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Quantile Regression [Koenker and Bassett, 1978]

A first idea is to use Quantile Regression :

$$q_\alpha^{QR}(Y|X=x) = \beta^{*\top} X + \beta_0^*,$$

where

$$(\beta^*, \beta_0^*) = \underset{\beta\in\mathbb{R}^N, \beta_0\in\mathbb{R}}{\arg\min}\ \mathbb{E}\left[\mathcal{S}_\alpha\left(Y - \beta^\top X - \beta_0\right)\right]$$

and

$$\mathcal{S}_\alpha(s) = (\alpha-1)s + \max\{s, 0\}.$$

Quantile Regression [Koenker and Bassett, 1978]

Theorem ([Maume-Deschamps et al., 2017a])

The quantile regression vector (β^*, β_0^*) *of* $Y|(X = x)$*, satisfying the previous problem, is given by*

$$\beta^* = \Sigma_X^{-1}\Sigma_{XY}, \quad \beta_0^* = \mu_Y - \Sigma_{YX}\Sigma_X^{-1}\mu_X + \sigma_{Y|X}\Phi_R^{-1}(\alpha).$$

The quantile regression predictor with level $\alpha \in [0, 1]$ *is given by*

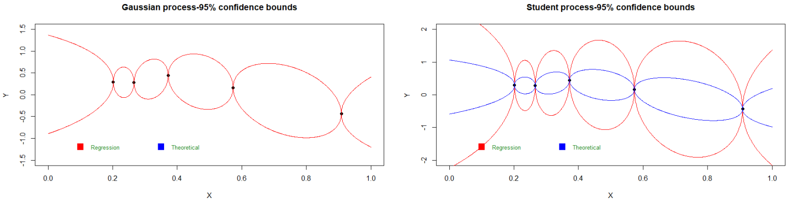
$$q_\alpha^{QR}(Y|X = x) = \mu_{Y|x} + \sigma_{Y|X}\Phi_R^{-1}(\alpha).$$

Furthermore, the distribution of $q_\alpha^{QR}(Y|X)$ *is*

$$q_\alpha^{QR}(Y|X) \sim \mathcal{E}_1\{\mu_Y + \sigma_{Y|X}\Phi_R^{-1}(\alpha), \Sigma_{YX}\Sigma_X^{-1}\Sigma_{XY}, \xi\}.$$

Comments

- Inefficient linear model.
- $\Phi_R^{-1}(\alpha)$ instead of $\Phi_{R^*}^{-1}(\alpha)$.



Some asymptotic relationships

Theorem ([Maume-Deschamps et al., 2017a])

Under some assumptions, there exist $0 < \ell < +\infty$ and $\eta \in \mathbb{R}$ such that :

$$\left[\Phi_R^{-1}\left(1 - \frac{1}{\frac{\ell}{1-\alpha}+2(1-\ell)}\right)\right]^{\frac{1}{\eta}} \underset{\alpha\to 1}{\sim} \Phi_{R^*}^{-1}(\alpha)$$

We thus define the following approximation for $q_\alpha(Y|X=x)$:

$$q_\alpha^E(Y|X=x) = \mu_{Y|x} + \sigma_{Y|X}\left[\Phi_R^{-1}\left(1 - \frac{1}{\frac{\ell}{1-\alpha}+2(1-\ell)}\right)\right]^{\frac{1}{\eta}}$$

Examples

Proposition ([Maume-Deschamps et al., 2017a])

The Gaussian, Student, Gaussian Mixture, and Slash distributions satisfy the previous assumptions, with coefficients η and ℓ given in the table below.

Distribution	η	ℓ
Gaussian	1	1
Student, $\nu > 0$	$\frac{N}{\nu}+1$	$\frac{\Gamma\left(\frac{\nu+N+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+N}{2}\right)\Gamma\left(\frac{\nu+1}{2}\right)}\left(1+\frac{q_1}{\nu}\right)^{\frac{N+\nu}{2}}\frac{\nu^{\frac{N}{2}+1}}{\nu+N}$
Gaussian Mixture	1	$\frac{\min(\theta_1^{-1},\dots,\theta_n^{-1})^N \exp\left(-\frac{\min(\theta_1^{-1},\dots,\theta_n^{-1})^2}{2} q_1\right)}{\sum_{k=1}^{n} \pi_k \theta_k^{-N} \exp\left(-\frac{1}{2\theta_k^2} q_1\right)}$
Slash, $a > 0$	$\frac{N}{a}+1$	$\frac{\Gamma\left(\frac{N+1+a}{2}\right) q_1^{\frac{N+a}{2}}}{\Gamma\left(\frac{N+a}{2}\right)(N+a)\chi^2_{N+a}(q_1) 2^{\frac{a}{2}-1}\Gamma\left(\frac{1+a}{2}\right)}$

Estimation ([Usseglio-Carleve, 2017])

- Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a random sample independant and identically distributed from an $(\xi, N+1)-$elliptical vector with (μ, Σ) known.

Assumption

We assume that there exist a function A such that $A(t) \to 0$ as $t \to +\infty$, and

$$\lim_{t \to +\infty} \frac{\frac{\Phi_R^{-1}\left(1-\frac{1}{\omega t}\right)}{\Phi_R^{-1}\left(1-\frac{1}{t}\right)} - \omega^\gamma}{A(t)} = \omega^\gamma \frac{\omega^\rho - 1}{\rho}$$

where $\gamma > 0$ and $\rho \leq 0$.

Parameters estimation

Proposition

Under the previous assumption, parameters η and ℓ exist, and are expressed :

$$\begin{cases} \eta = & 1 + \gamma N \\ \ell = & \dfrac{\Gamma\left(\frac{N+\gamma^{-1}+1}{2}\right)}{\Gamma\left(\frac{\gamma^{-1}+1}{2}\right)} \dfrac{\gamma^{-1}\pi^{-\frac{N}{2}}}{(N+\gamma^{-1})c_N g_N(m(x))}. \end{cases}$$

where $m(x) = (x - \mu_X)^T \Sigma_X^{-1} (x - \mu_X)$.

Parameters estimation

Definition

We define the two following estimators :

$$\begin{cases} \hat{\eta}_{k_n} = & N\hat{\gamma}_{k_n} + 1 \\ \hat{\ell}_{k_n,h_n} = & \dfrac{\Gamma\left(\frac{N+\hat{\gamma}_{k_n}^{-1}+1}{2}\right)}{\Gamma\left(\frac{\hat{\gamma}_{k_n}^{-1}+1}{2}\right)} \dfrac{\hat{\gamma}_{k_n}^{-1}\pi^{-\frac{N}{2}}}{\left(N+\hat{\gamma}_{k_n}^{-1}\right)\frac{m(x)^{1-\frac{N}{2}}\Gamma\left(\frac{N}{2}\right)}{\pi^{\frac{N}{2}}nh_n}\sum_{i=1}^{n}K\left(\frac{m(x)-(X_i-\mu_X)^T\Sigma_X^{-1}(X_i-\mu_X)}{h_n}\right)} \end{cases},$$

where $\hat{\gamma}_{k_n} = \frac{1}{k_n}\sum_{i=1}^{k_n}\ln\left(\frac{W_{[i]}}{W_{[k_n+1]}}\right)$, $k_n = o(n)$, $h_n = o(1)$, $k_n \to +\infty$, $nh_n \to +\infty$ as $n \to +\infty$ and W is the first (or indifferently any) component of the vector $\Lambda_X^{-1}(X - \mu_X)$.

Parameters estimation

Proposition

Under our assumption, and if $\sqrt{k_n}A\left(\frac{n}{k_n}\right) \to 0$ *as* $n \to +\infty$*, then the following asymptotic relationships hold :*

- *If* $nh_n/k_n \underset{n\to+\infty}{\to} 0$*, then*

$$\sqrt{nh_n}\begin{pmatrix}\hat{\ell}_{k_n,h_n} - \ell \\ \hat{\eta}_{k_n} - \eta\end{pmatrix} \underset{n\to+\infty}{\sim} \mathcal{N}\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}V_2 & 0\\ 0 & 0\end{pmatrix}\right)$$

where $V_2 =$

$$\frac{\Gamma\left(\frac{N}{2}\right)}{m(x)^{\frac{N}{2}-1}\pi^{\frac{N}{2}}} c_N g_N(m(x)) \int K(u)^2 du \left[\frac{\Gamma\left(\frac{N+\gamma^{-1}+1}{2}\right)}{\Gamma\left(\frac{\gamma^{-1}+1}{2}\right)} \frac{\left(N+\gamma^{-1}\right)^{-1}\gamma^{-1}\pi^{-\frac{N}{2}}}{c_N^2 g_N(m(x))^2}\right]^2$$

Parameters estimation

Proposition

Under our assumption, and if $\sqrt{k_n} A\left(\frac{n}{k_n}\right) \to 0$ *as* $n \to +\infty$*, then the following asymptotic relationships hold :*

- *If* $nh_n/k_n \underset{n\to+\infty}{\to} +\infty$*, then*

$$\sqrt{k_n}\begin{pmatrix}\hat{\ell}_{k_n,h_n} - \ell \\ \hat{\eta}_{k_n} - \eta\end{pmatrix} \underset{n\to+\infty}{\sim} \mathcal{N}\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix} V_1 & -N\gamma\sqrt{V_1} \\ -N\gamma\sqrt{V_1} & N^2\gamma^2\end{pmatrix}\right)$$

where $V_1 =$

$$\frac{\pi^{-N}\gamma^2}{c_N^2 g_N(m(x))^2}\,\frac{\Gamma\left(\frac{N+\gamma^{-1}+1}{2}\right)^2}{\Gamma\left(\frac{\gamma^{-1}+1}{2}\right)^2}\left[\frac{\Psi\left(\frac{\gamma^{-1}+1}{2}\right) - \Psi\left(\frac{N+\gamma^{-1}+1}{2}\right)}{2\gamma^2\left(N\gamma+1\right)} - \frac{N}{(N\gamma+1)^2}\right]^2$$

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Parameters estimation

Proposition

Under our assumption, and if $\sqrt{k_n}A\left(\frac{n}{k_n}\right) \to 0$ *as* $n \to +\infty$, *then the following asymptotic relationships hold :*

- *If* $nh_n/k_n \underset{n\to+\infty}{\longrightarrow} c \in \mathbb{R}_+^*$, *then*

$$\sqrt{k_n}\begin{pmatrix}\hat{\ell}_{k_n,h_n} - \ell \\ \hat{\eta}_{k_n} - \eta\end{pmatrix} \underset{n\to+\infty}{\sim} \mathcal{N}\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}V_1 + \frac{1}{c}V_2 & -N\gamma\sqrt{V_1} \\ -N\gamma\sqrt{V_1} & N^2\gamma^2\end{pmatrix}\right)$$

Quantile estimation

We consider $(\alpha_n)_n$ such that $\alpha_n \to 1$ as $n \to +\infty$, and distinguish two cases :

- Intermediate quantiles, i.e we suppose $n(1-\alpha_n) \to +\infty$. It entails that the estimation of the α_n-quantile leads to an interpolation of sample results.
- High quantiles. We suppose $n(1-\alpha_n) \to 0$, i.e we need to extrapolate sample results to areas where no data are observed.

Quantile estimation

Definition

We define the two following estimators, respectively for intermediate and high quantiles :

$$\begin{cases} \hat{q}^{E}_{\alpha_n}(Y|X=x) = & \mu_{Y|x} + \sigma_{Y|X}\left(W_{[\tilde{k}_n+1]}\right)^{\frac{1}{\hat{\eta}_{k_n}}} \\ \hat{\hat{q}}^{E}_{\alpha_n}(Y|X=x) = & \mu_{Y|x} + \sigma_{Y|X}\left[W_{[k_n+1]}\left(\frac{k_n}{n}\left(2+\hat{\ell}_{k_n,h_n}\left(\frac{1}{1-\alpha_n}-2\right)\right)\right)^{\hat{\gamma}_{k_n}}\right]^{\frac{1}{\hat{\eta}_{k_n}}} \end{cases},$$

where $\tilde{k}_n = \frac{n}{2+\hat{\ell}_{k_n,h_n}\left(\frac{1}{1-\alpha_n}-2\right)}$ and W is the first (or indifferently any) component of the vector $\Lambda_X^{-1}(X-\mu_X)$.

Quantile estimation

Theorem

Consider that our assumption holds and assume that :

- $k_n = o(nh_n)$ *and* $\sqrt{k_n} A\left(\frac{n}{k_n}\right) \to 0$ *as* $n \to +\infty$.
- $n(1-\alpha_n) \to +\infty$, $\ln(1-\alpha_n) = o(\sqrt{k_n})$ *and* $\frac{\sqrt{k_n}}{\ln(1-\alpha_n)} = o\left(\sqrt{n(1-\alpha_n)}\right)$ *as* $n \to +\infty$.

Then

$$\frac{\sqrt{k_n}}{\ln\left(1-\alpha_n\right)}\left(\frac{\hat{q}^E_{\alpha_n}(Y|X=x)}{q^E_{\alpha_n}(Y|X=x)} - 1\right) \underset{n\to+\infty}{\sim} \mathcal{N}\left(0, \frac{N^2\gamma^4}{(\gamma N+1)^4}\right).$$

Therefore :

$$\frac{\hat{q}^E_{\alpha_n}(Y|X=x)}{q_{\alpha_n}(Y|X=x)} \overset{\mathbb{P}}{\to} 1 \text{ as } n \to +\infty.$$

Quantile estimation

Theorem

We denote $p_n = \left(2+\ell\left((1-\alpha_n)^{-1}-2\right)\right)^{-1}$, $\tilde{p}_n = \left(2+\hat{\ell}_{k_n,h_n}\left((1-\alpha_n)^{-1}-2\right)\right)^{-1}$.
Consider that our assumption holds and assume that :

- $k_n = o(nh_n)$ *and* $\sqrt{k_n}A\left(\frac{n}{k_n}\right) \to 0$ *as* $n \to +\infty$.
- $n(1-\alpha_n) \to 0$, $\ln\left(n(1-\alpha_n)\right) = o(\sqrt{k_n})$ *and* $\frac{\ln(1-\alpha_n)}{\ln\left(\frac{n}{k_n}(1-\alpha_n)\right)} \to \theta \in [0,+\infty[$.

Then

$$\frac{\sqrt{k_n}}{\ln\left(\frac{k_n}{n(1-\alpha_n)}\right)}\left(\frac{\hat{\hat{q}}^E_{\alpha_n}(Y|X=x)}{q^E_{\alpha_n}(Y|X=x)}-1\right) \underset{n\to+\infty}{\sim} \mathcal{N}\left(0,\left(\frac{\gamma}{\gamma N+1}-\theta\frac{\gamma^2 N}{(\gamma N+1)^2}\right)^2\right)$$

Therefore :

$$\frac{\hat{\hat{q}}^E_{\alpha_n}(Y|X=x)}{q_{\alpha_n}(Y|X=x)} \xrightarrow{\mathbb{P}} 1 \text{ as } n \to +\infty.$$

L_p-quantiles

Definition ([Chen, 1996])

Let Z be a real random variable. The L_p-quantiles of Z with level $\alpha \in]0,1[$ and $p > 0$, denoted $q_{p,\alpha}(Z)$, is solution of the minimization problem :

$$q_{p,\alpha}(Z) = \arg\min_{z\in\mathbb{R}} \mathbb{E}\left[(1-\alpha)(z-Z)_+^p + \alpha(Z-z)_+^p\right]$$

where $Z_+ = Z\mathbb{1}_{\{Z>0\}}$.

L_p-quantiles

- According to [Koenker and Bassett, 1978], the case $p = 1$ leads to the quantile $q_{1,\alpha}(Z) = F_Z^{-1}(\alpha)$.
- The case $p = 2$ is called expectile, which knows a growing interest : [Taylor, 2008], [Cai and Weng, 2016], [Maume-Deschamps et al., 2017b].
- Other cases are difficult to deal with : [Bernardi et al., 2017].

Extreme L_p−quantiles

Theorem ([Daouia et al., 2017])

If a random variable Z fills our assumption, then

$$\frac{q_{p,\alpha}(Z)}{q_\alpha(Z)} \underset{\alpha \to 1}{\sim} \left[\frac{\gamma}{\beta\left(p, \gamma^{-1} - p + 1\right)}\right]^{-\gamma} = f_L(\gamma, p)$$

where β is the beta function.

Lemma

Under our assumption, the conditional distribution $Y|X = x$ is attracted to a maximum domain of Pareto-type distribution with tail index $(\gamma^{-1} + N)^{-1}$.

Conditional extreme L_p-quantiles estimation

Definition

Let $(\alpha_n)_{n\in\mathbb{N}}$ be a sequence such that $\alpha_n \to 1$ as $n \to +\infty$. We define :

$$\begin{cases} \hat{q}_{p,\alpha_n}(Y|X=x) = & \mu_{Y|x} + \sigma_{Y|X}\left(W_{[n\tilde{p}_n+1]}\right)^{\frac{1}{\hat{\eta}_{k_n}}} f_L\left(\left(\hat{\gamma}_{k_n}^{-1}+N\right)^{-1}, p\right) \\ \hat{\hat{q}}_{p,\alpha_n}(Y|X=x) = & \mu_{Y|x} + \sigma_{Y|X}\left[W_{[k_n+1]}\left(\frac{k_n}{n}\left(2+\hat{\ell}_{k_n,h_n}\left(\frac{1}{1-\alpha_n}-2\right)\right)\right)^{\hat{\gamma}_{k_n}}\right]^{\frac{1}{\hat{\eta}_{k_n}}} \\ & \times f_L\left(\left(\hat{\gamma}_{k_n}^{-1}+N\right)^{-1}, p\right) \end{cases}$$

Haezendonck-Goovaerts risk measures

Definition ([Haezendonck and Goovaerts, 1982])

Let Z be a real random variable, and φ a non negative and convex function with $\varphi(0) = 0$, $\varphi(1) = 1$ and $\varphi(+\infty) = +\infty$. The Haezendonck-Goovaerts risk measure of Z with level $\alpha \in]0,1[$ associated to φ, is given by the following :

$$H_\alpha(Z) = \inf_{z \in \mathbb{R}} \{z + H_\alpha(Z,z)\}$$

where $H_\alpha(Z,z)$ is the unique solution h to the equation :

$$\mathbb{E}\left[\varphi\left(\frac{(Z-z)_+}{h}\right)\right] = 1 - \alpha$$

φ is called Young function.

Extreme Haezendonck-Goovaerts risk measures

- The case $\varphi(t) = t$ leads to the Tail Value-at-Risk $\text{TVaR}_\alpha(Z)$.

Proposition ([Tang and Yang, 2012])

If Z fills our assumption, and taking a Young function $\varphi(t) = t^p, p \geq 1$, then the following relationship holds :

$$\frac{H_{p,\alpha}(Z)}{q_\alpha(Z)} \underset{\alpha \to 1}{\sim} \frac{\gamma^{-1}\left(\gamma^{-1} - p\right)^{p\gamma - 1}}{p^{\gamma(p-1)}} \beta\left(\gamma^{-1} - p, p\right)^{\gamma} = f_H(\gamma, p)$$

Conditional extreme H-G risk measures estimation

Definition

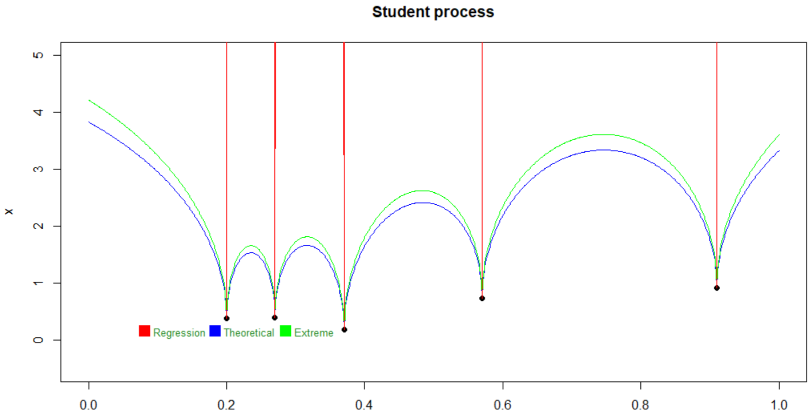
Let $(\alpha_n)_{n\in\mathbb{N}}$ be a sequence such that $\alpha_n \to 1$ as $n \to +\infty$. We define :

$$\begin{cases} \hat{H}_{p,\alpha_n}(Y|X=x) = & \mu_{Y|x} + \sqrt{\Sigma_{Y|X}} \left(W_{[n\tilde{p}_n+1]}\right)^{\frac{1}{\hat{\eta}_{k_n}}} f_H\left(\left(\hat{\gamma}_{k_n}^{-1}+N\right)^{-1}, p\right) \\ \hat{\hat{H}}_{p,\alpha_n}(Y|X=x) = & \mu_{Y|x} + \sqrt{\Sigma_{Y|X}} \left[W_{[k_n+1]}\left(\frac{k_n}{n}\left(2+\hat{\ell}_{k_n,h_n}\left(\frac{1}{1-\alpha_n}-2\right)\right)\right)^{\hat{\gamma}_{k_n}}\right]^{\frac{1}{\hat{\eta}_{k_n}}} \\ & \times f_H\left(\left(\hat{\gamma}_{k_n}^{-1}+N\right)^{-1}, p\right) \end{cases}$$

Simulation study

- We apply our estimators to a sample of a centered Student process with $\nu = 1.5$ degrees of freedom, and compare with theoretical results.
- $\sigma(t) = e^{-|t|}$.
- We take the sequences $k_n = n^{0.6}$, $h_n = n^{-0.15}$ and $\alpha_n = 1 - n^{-1.1}$.
- The chosen kernel K is the gaussian p.d.f.
- Theoretical result is $q_\alpha(Y|X=x) = \sqrt{\frac{\nu+m(x)}{\nu+N}}\Phi^{-1}_{\nu+N}(\alpha)$

Simulation study



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Merci !