# Stochastic Inversion Under Functional Uncertainties [Poster]

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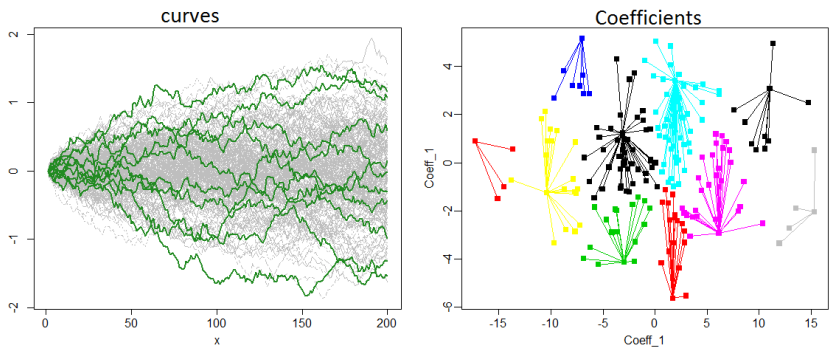
**Abstract:**

In the present work, we consider a system that evolves in an uncertain environment. The considered system, denoted $f$, takes two types of input variables, a set of control variables $x \in \mathrm{X}$, and a set of uncertain variables $v \in \mathcal{V}$. Robust inversion consists in seeking the set of control variables $x \in \mathrm{X}$ such that $\sup_{v \in \mathcal{V}} f(x, v) \leq c$, with $c \in \mathrm{R}$ a prescribed threshold. The difficulty of solving the robust inversion problem strongly depends on the uncertainty set $\mathcal{V}$. In our case, $\mathcal{V}$ is a functional space, and we consider the inversion problem under uncertainty as a stochastic inversion problem.

Let V denote the random variable, valued in $\mathcal{V}$, modeling the uncertainty. In the following, we are interested in the set : $\Gamma^* := \{x \in \mathrm{X} \ , \ g(x) = \mathrm{E_V}[f(x, \mathrm{V})] \leq c\}$. In our framework, the probability distribution of V is only known from a set of realizations. We thus aim at replacing the expectation in the definition of $\Gamma^*$ by a Monte Carlo estimate. In that sens, our methodology is a data-driven procedure.

There are two major methods to model uncertainties associated with functional random variables [3]. The first one, FPCA, relies on the theory of functional analysis and consists in two steps : 1) Developing the functional random variable on a truncated basis based on Karhunen-Loève decomposition; 2) modeling and inferring the probability distribution of the vector of coefficients resulting from the truncated decomposition. The second one, Scenario modeling, is based on a discrete approximation of the functional distribution. Those two methods suffer from various issues. The first method depends on the strategy used for the estimation of the coefficients distribution and another disadvantage is that the order of truncation may be needed large to preserve the underlying curve structure. The second approach has the merit of keeping the shape of curves, but the realizations can only be randomly selected from the initial set. Therefore, we propose a new approach, called weighted scenario, mixing good properties of Space Filling Design and Karhunen-Loève expansion, to explain at best the variability of V with a reduced sample of realizations (figure below).



In this work, we adopt a sequential sampling strategy based on Gaussian process emulators. The idea is that gaussian process emulators, which capture prior knowledge about the regularity of the unknown function $g : x \mapsto \mathrm{E}_{\mathrm{V}}[f(x, \mathrm{V})]$, make it possible to assess the uncertainty on $\Gamma^*$ given a set of evaluations. More specifically, we focus on the Stepwise Uncertainty Reduction (SUR) infill sampling criterion. This criterion was originally introduced in the context of inversion problem in [1]. Briefly, the strategy SUR gives sequentially the next location where the function $g$ should be evaluated in order to minimize an uncertainty function. The choice of the uncertainty function is based on the theory of random closed sets [2]. The key contribution of the present work is to propose a data-driven adaptation of that procedure in the presence of functional uncertainties. We conducted numerical experimentations on an analytical test case to compare the performance of Scenario modelling, FPCA modelling and Weighted Scenario. The figure below shows the set inferred after 20 added points to an initial DoE of 9 points, where the uncertainties come from a standard Brownian Motion with a state space in R, and only known from a set of 200 realizations. At each iteration, i.e. added point, the expectation is estimated sequentially by referring to a stopping criteria in order to check the stability of the estimate. We remark the efficiency of our method to estimate the set $\Gamma^*$ with small number of calls to the function $f$.

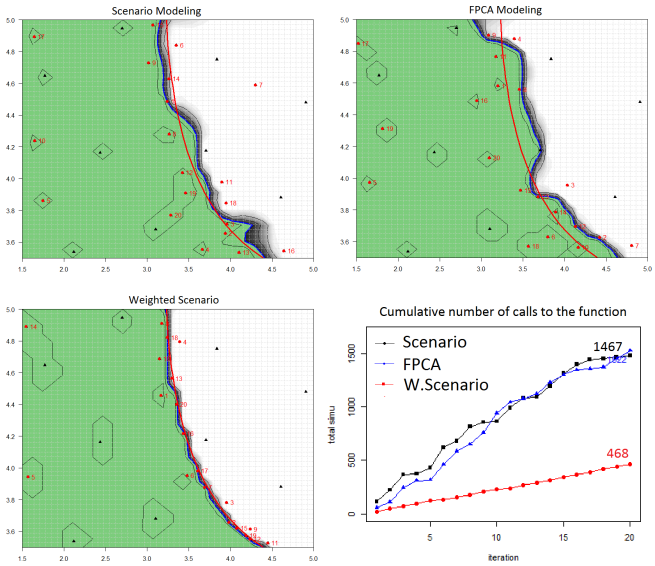



Figure 1: Initial points (black triangles), the 20 added points (red points). Boundary of the true set (red). Boundaries of the estimate sets (dark blue). Cumulative number of calls to the function (bottom right)

## References

[1] Clément Chevalier. *Fast uncertainty reduction strategies relying on Gaussian process models.* PhD thesis, Citeseer, 2013.

[2] Ilya Molchanov. *Theory of random sets.* Springer Science & Business Media, 2006.

[3] Simon Nanty. *Quantification des incertitudes et analyse de sensibilité pour codes de calcul à entrées fonctionnelles et dépendantes.* PhD thesis, Université Grenoble Alpes, 2015.

**Short biography** – I obtained a MSc in Applied mathematics : Computer and Stochastic Methods for Decision in 2015 at Université de Pau et des Pays de l'Adour. I started my PhD with IFPEN and University Grenoble Alpes in April 2016 in the OQUAIDO project (Optimisation et QUAntification d'Incertitudes pour les Données Onéreuses).