# Stochastic metamodeling applied to dosimetry

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**Abstract:**

Stochastic city models were applied in the previous study [1] to assess the global Electromagnetic Field (EMF) exposure of a population. These stochastic models were constructed in terms of the main structures of a city (e.g., street length and orientation, building high). In other words, the same input leads to different outputs. Today, large efforts have been dedicated to surrogate deterministic simulators, it is therefore in our interest to develop methods dedicated to the stochastic ones. This paper proposes an innovative approach to built a surrogate model for stochastic simulators.

# 1 Context

Previous studies [1] have assessed the EMF global exposure but the method used was time consuming. The objective is therefore to simplify the approach using simplified propagation model combined with the macroscopic characteristics (street length, anisotropy..) of the city [2]. In this case the received power can be modeled as a function of the distance from the receiver to the transmitter weighted by a path loss coefficient (PLE) wich is affected by the morphological parameters of the city. As shown in [2], the city's parameters can induce different PLE. So PLE can be seen as a stochastic process (SP). The assessment of PLE distribution requests huge computational time, it is therefore of interest to look for a surrogate modeling of stochastic process.
Surrogate modeling of deterministic simulators is a quite mature fields. In contrast, the question of surrogating stochastic simulators has arisen only recently in the literature e.g. [3]. The surrogate modeling of SP can be decomposed in two classes. The first one, i.e. parametric, considers that the SP comes from a known distribution. The surrogate modeling of SP can be then seen as the surrogate modeling of the parameters of the distribution. The second one, i.e. not parametric, do not put any hypothesis on the distribution of the SP. The latest approach is considered in this work. This work will first investigate the use of Karhunen-Loève for the analysis of SP. We will after that look at the surrogate modeling of a correlation matrix to assess more accurately the SP. At the end the results will be concluded.

# 2 Stochastic process modeling

Let $\mathcal{H}(t,\omega)$ be a random process of second order with zero mean, where $t$ is a deterministic vector and $\omega$ belonging to the space of random events $\Omega$, and $C(s,t)$ its covariance, this approach represent a metamodel based on the Karhunén loéve decomposition.

## 2.1 Karhunen-Loève Expansion

Karhunen-Loève (KL) expansion [4] is a spectral decomposition in a infinite linear combination of orthogonal functions as:

$$\mathcal{H}(t,\omega) = \lim_{p\to\infty} \sum_{i=1}^{p} \sqrt{\lambda_i}\xi_i(\omega)\phi_i(t)$$

where $\{\xi_i(\omega),\ i \in \mathbb{N}\}$ is a set of random variables to be determined, $\lambda_n$ and $\phi_n$ are, respectively, eigenvalues and eigenvectors of $C(s,t)$.

### 2.2 Surrogate modeling of a stochastic process

Traditionally KL decomposition is used to simulate and represent a stochastic process analogous to Fourier series representation of a function. In this work, KL is rather used to surrogate $\mathcal{H}$. As a first step we run simulations to build a numerical covariance of $\mathcal{H}$ denoted $\hat{C}$. Eigendecomposition of $\hat{C}$ was then carried out. The $\mathcal{H}$ surrogate, i.e. $\hat{\mathcal{H}}$, can be written as follows:

$$\hat{\mathcal{H}}(t,\omega) = \sum_{i=1}^{N} \sqrt{\lambda_i}\xi_i(\omega)\phi_i(t)$$

Where $\xi_i$ are orthogonal Gaussian variables.

### 2.3 Surrogate model of the covariance

Stochastic simulators are often time consuming, in our context one run takes 3 hours. Building the covariance matrix requires $N.M$ runs, $N$ is the number of points whereas $M$ denotes the realizations on each point. That is why we need to surrogate the covariance. Deterministic approaches like Kriging, Polynomial Chaos (PC) interpolation or support vector machines (SVM) can be carried to surrogate $\hat{C}$. Let $\tilde{C}$ be the PC surrogate of $\hat{C}$, thus KL decomposition will be carried on $\tilde{C}$ instead of $\hat{C}$.

### 2.4 Example

We consider a simulated Gaussian process defined on $[0,1] \times \Omega$ and the corresponding covariance $\hat{C}$ on $N$ points. A PC surrogate model of $\hat{C}$ is constructed, the KL decomposition is then applied to surrogate the SP. On a given new point $t^*$ we can predict the probability density function as in Fig.1.

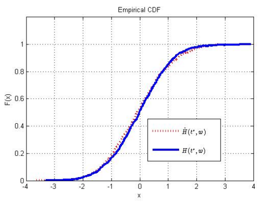


Figure 1: *Comparison of empirical Cumulative Density Function (blue curve) with the surrogated one (red dashed curve)*

## 3 Conclusion

This work proposed an innovative approach of stochastic metamodeling using deterministic metamodeling approaches with KL spectral decomposition. The accuracy of the metamodel depends on $N$ and $M$ as well as the precision of the expansions.

**References**

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**Short biography** – Azzi Soumaya is a Ph.D. student within Chaire C2M, LTCI, Télécom ParisTech. Her research interests include stochastic computation, surrogate modeling and uncertainty quantification. She received the M.Sc. degree in applied mathematics from Blaise Pascal University, Clermont F$^{rd}$, France.