Reliability-based sensitivity analysis under distribution parameter uncertainty – Application to aerospace systems

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Abstract:

Assessing reliability of aerospace systems relies on the use of complex computer codes whose aim is to mimick the behavior of the real system under study (e.g., fallback phase estimation of a space vehicle, collision between a satellite and space debris). Such a simulation framework can be highly nonlinear and/or expensive-to-evaluate, and possibly mixing multi-physics inner codes. As a consequence, it is often modeled as a black-box function only known pointwise. Uncertainty quantification (UQ) thus plays a major role from both the design and the certification of these systems.

From the UQ point of view, one needs first to model the uncertainties arising from several sources in input of the computer code. In a probabilistic framework, the phenomenological uncertainties [1] can be modeled by assuming that input variables are a set of random variables gathered in the vector $\mathbf{X}$ following some probability distribution given by a parametric probability density function (pdf) $f_{\mathbf{X}|\boldsymbol{\Theta}}(\mathbf{x}|\boldsymbol{\theta})$. However, the distribution parameters $\boldsymbol{\theta}$ can be affected by uncertainty arising from lack of data (i.e. statistical uncertainty due to limited amount of data) or resting on expert recommendations. Consequently, given a failure scenario, reliability measures (e.g., failure probability, reliability index) which are estimated regarding the distribution of the output should take this second uncertainty level into account. This bi-level uncertainty can be tackled and formalized either within the probabilistic Bayesian framework [2] or with the so-called "imprecise probabilistic" one [3].

In the present work, the Bayesian framework is adopted by considering that the uncertainty affecting distribution parameters can be modeled using a prior distribution $f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}|\boldsymbol{\xi})$ for $\boldsymbol{\Theta}$, now considered as a random vector. Let $\boldsymbol{\xi}$ denote the vector of deterministic hyper-parameters encoding the prior knowledge. One can thus propagate the bi-level uncertainty contained in $(\mathbf{X}, \boldsymbol{\Theta})$. The so-called "*predictive failure probability*" (PFP) which is the mean estimator of all the failure probabilities regarding the variability of the distribution parameters [4] can be a relevant measure of safety for design purposes as it takes into account both levels of uncertainty:

$$\widetilde{P}_{\mathrm{f}} = \mathbb{E}_{f_{\boldsymbol{\Theta}}}\left[P_{\mathrm{f}}(\boldsymbol{\Theta})\right] = \int_{D_{\boldsymbol{\Theta}}} P_{\mathrm{f}}(\boldsymbol{\theta}) f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} = \int_{D_{\boldsymbol{\Theta}}} \left(\int_{D_{\mathbf{X}}} \mathbf{1}_{F_{\mathbf{x}}}(\mathbf{x}) f_{\mathbf{X}|\boldsymbol{\Theta}}(\mathbf{x}|\boldsymbol{\theta})\mathrm{d}\mathbf{x} \right) f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}|\boldsymbol{\xi})\mathrm{d}\boldsymbol{\theta}. \quad (1)$$

The first contribution in this work consists in showing that this PFP can be estimated in an efficient way by considering an augmented framework, i.e. by estimating the double integral using the joint vector $\mathbf{Z} = (\mathbf{X}, \boldsymbol{\Theta})^\top$ and by adapting the Rosenblatt transformation in the advanced rare event sampling methods (e.g., importance sampling, subset simulations) [5].

The second contribution consists in proposing a local derivative-based sensitivity estimator for this PFP with respect to distribution hyper-parameters [6]. Indeed, if getting a PFP estimate is crucial from the reliability assessment point of view, estimating the sensitivity of this quantity of interest regarding the chosen values for the deterministic parameters $\xi_j \in \boldsymbol{\xi}$ can be relevant for screening purposes (i.e. to set unessential distribution parameters regarding their influence on the PFP). To do so, one took advantage of the preliminary PFP estimation phase. Two cases are treated: either the prior distribution lies over an unbounded support or a bounded one.

Thus, the third contribution of this work is to propose an adaptive importance sampling-based strategy, within the augmented space, to estimate both the PFP and its local sensitivities to the a priori parametric choice in $\boldsymbol{\xi}$. Thus, considering the case where the prior $f_{\boldsymbol{\Theta}}(\cdot|\cdot)$ lies over an unbounded support, one gets:

$$\frac{\partial \widetilde{P}_{\mathrm{f}}(\boldsymbol{\xi})}{\partial \xi_j} = \int_{D_{\mathbf{Z}}} \mathbf{1}_{F_{\mathbf{z}}}(\mathbf{z}) s_j(\boldsymbol{\theta}, \boldsymbol{\xi}) w(\mathbf{z}) h_{\mathbf{Z}}(\mathbf{z}) \mathrm{d}\mathbf{z} \tag{2}$$

where $s_j(\boldsymbol{\theta}, \boldsymbol{\xi}) = \frac{\partial \ln f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}|\boldsymbol{\xi})}{\partial \xi_j}$ is called the "*score function*" and $w(\mathbf{z}) = \frac{f_{\mathbf{Z}}(\mathbf{z}|\boldsymbol{\xi})}{h_{\mathbf{Z}}(\mathbf{z})}$ is the likelihood ratio arising from the importance sampling with auxiliary density $h_{\mathbf{Z}}(\cdot)$ in the augmented space. In the unbounded case, getting the local sensitivity of the PFP is just a post-treatment of the rare event estimation and thus requires no extra computational effort in terms of code simulation. Sensitivity estimators in the bounded case have also been proposed in [7]. The efficiency of the method has been assessed on both academic and real aerospace test cases.

Some perspectives could be to extend this work in the augmented space to estimate global sensitivity indices taking the bi-level uncertainty into account, over the failure domain, and to estimate them with a reduced number of calls to the black-box function.

References

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Short biography – V. Chabridon got his Engineer's degree from the French Institute for Advanced Mechanics (IFMA) and a Master degree in Materials and Structural Reliability from Université Clermont Auvergne. He is currently enrolled in a PhD program co-funded by ONERA and SIGMA Clermont (ex-IFMA). His topic focuses on the uncertainty propagation and reliability-based sensitivity analysis with applications to aerospace systems reliability assessment.