MascotNum2018 conference - Shapley effects for sensitivity analysis with dependent inputs

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Abstract:

The global sensitivity analysis of a numerical model, $Y = f(\mathbf{X})$ with d random inputs $\mathbf{X} = (X_1, \ldots, X_d)$, consists in quantifying the contributions of each of its input parameters in the variability of its output. Based on the functional ANalyse Of VAriance (ANOVA), the popular Sobol' indices [8, 1] present a faulty interpretation in the presence of **dependence** between inputs. Several works have been carried to deal with this difficulty and extend Sobol' indices to the case of a stochastic dependence between the input variables, as [3, 5]. But the practical estimation of these sensitivity measures and their interpretation remains difficult. Recently introduced in this context, the Shapley effects [7] coming from the field of game theory consist of allocating a part of the variance of the output at each input, make it possible to solve this problem.

[9] proposed an algorithm to estimate the Shapley effects. Some studies highlighted the potential of this kind of index in the case of correlated input, as [6, 2]. In this last case the Shapley effects can be a good alternative to the existing extensions of Sobol' indices mentioned above. Indeed, Shapley effects allows an apportionment of the interaction and dependances contributions between the input involved, making them condensed and easy-to-interpret indices.

Most estimation procedures of the Sobol' and Shapley indices are based on Monte-Carlo sampling. These methods require large sample sizes in order to have a sufficiently low estimation error. When dealing with costly computational models, a precise estimation of these indices can be difficult to achieve or even unfeasible. Therefore, the use of a surrogate model (or meta-model) instead of the actual one can be a good alternative and dramatically decrease the computational cost of the estimation. Various kinds of surrogate model exists in the literature and we get interested in the use of Kriging models also called Gaussian processes regression (GP) as meta-models.

Hence, ones make a comparison between the Shapley effects and the *independent* and *full* Sobol' indices defined in [5]. Based on [4], we also develop an estimation algorithm of the Shapley effects using Kriging models which allows to catch and compute the meta-model and Monte-Carlo errors.

References

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Short biography – I obtained a Master of Research in mathematical engineering. Since May 2017, I had the chance to start my PhD under CIFRE convention with SCOR SE. The goal of this PhD is to develop of new quantitative tools for model risk and the taking into account of uncertainties, in particular for the calculation of the Solvency Capital Requirement (SCR).