Polynomial chaos expansion for acoustic propagation

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In the atmosphere, under particular conditions, an acoustic signal can propagate over large distance. Alongside this propagation the signal can be severly distorted but normal mode decomposition allows a quite accurate simulation, given the atmospheric conditions.
However, uncertainties on the atmospheric conditions can dramatically change the propagation. The purpose of this work is to link those uncertainties with the form of the receipt signal in the particular case of nocturnal boundary layer propagation [4].
This configuration occurs during clear nights when there is a temperature inversion at the ground which creates a strong duct for acoustic propagation. Above the inversion, a strong geostrophic wind, also called « nocturnal jet », can accentuate this waveguide, its characteristics will be considered uncertain.

Normal mode decomposition

In the linear acoustic approximation, the field of pressure satisfies the wave equation which can be solved by a modal decomposition.
The decomposition consists in computing the eigenvalues $(k_j(\omega))_{j\in J}$ and eigenfunctions $(\Psi_j(\omega,z))_{j\in J}$ of the propagation operator: $\Psi_{zz} + \omega^2\Psi/c(z)^2 = k\Psi$ where $c(z)$ is the effective sound speed.
Those eigenpairs allows to compute the sum of residuals :

$$G(\omega,R) = \frac{e^{\frac{i\pi}{4}}}{\sqrt{8\pi}} \sum_{j\in J} \frac{\Psi_j(\omega,0)^2}{\sqrt{k_j(\omega)R}} e^{ik_j(\omega)R}$$

Then, the Fourier transform of the pressure at range R will be given by: $\hat{p}(\omega;R) = G(\omega;R)\hat{s}(\omega)$ where $\hat{s}$ is the Fourier transform of the source signal.

Polynomial chaos expansion

The atmospheric conditions depend on uncertain parameters $\xi = (\xi_i)$ which impact the propagation of acoustic waves. To build a metamodel of the propagation taking into account those uncertainties, a non-intrusive polynomial chaos expansion has been considered [2] using quadrature-based formulae for computing the gPC coefficients.

In order to have a metamodel independant from the distance and the source, expansions of the eigenvalues $\widehat{k_j}(\omega,\xi)$ and eigenvectors $\widehat{\Psi_j}(\omega,\xi)$ has been considered. Then, one can chose a distance R, compute $\widehat{G}(\omega,R,\xi)$ and use it to generate a signal for any realisation of the perturbation and any source signal.

This method gives a metamodel for each eigenvalue and eigenfunction but each projection uses the same quadrature points and therefore it has the same computational cost as a classical non intrusive method.

Moreover, this strategy allows to avoid a classical problem with gPC expansion: the « long-term integration problem ». Uncertainties affect the phase velocity and the complexity of the system increases with distance precluding accurate gPC approximation [3]. But eigenpairs are independant from the distance and thereby our gPC expansions are not affected.

The eigenvalues and eigenfunctions are defined for frequencies above a certain value $\omega_j(\xi)$ depending on the celerity profil and thereby on the uncertainties. To avoid discontinuities, a continuation of eigenvalues and eigenfunctions has been used for gPC projection. The sum of residuals is then multiplied by an indicator function $\mathbf{1}_{[\widehat{\omega_j}(\xi);\omega_{max}]}(\omega)$ where $\widehat{\omega_j}(\xi)$ is a gPC projection of $\omega_j(\xi)$.

One of the key issue of this method is to follow the eigenvalues despite the perturbations in order to calibrate the metamodel of a given eigenvalue. The eigenfunctions have been used: from one point of quadrature to the next an eigenfunction is linked with the one maximizing their L^1 scalar product.

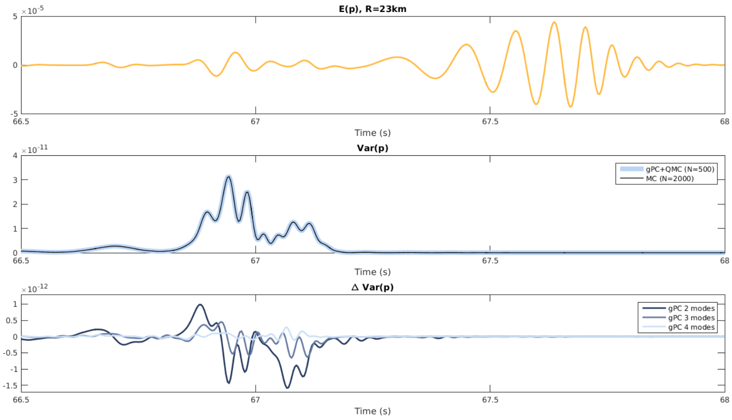

Figure 1: Example of signal at 23.2km from the source (top). Variance computed with the gPC metamodel and with monte Carlo simulations (middle). Convergence of the gPC expansion (bottom). Here the uncertainties are on the jet and impact only the fastest acoustic modes hence the absence of variance on the second arrival.

References

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Short biography – This PhD, started in January 2017, at ENS Paris-Saclay is funded by CEA. CEA is working with the CTBTO (Comprehensive Nuclear Test Ban Treaty Organization) on the detection of explosions on the surface of the globe using infrasound monitoring stations.