Uncertainty quantification in large systems of solvers: application to reentering man-made space object trajectory prediction

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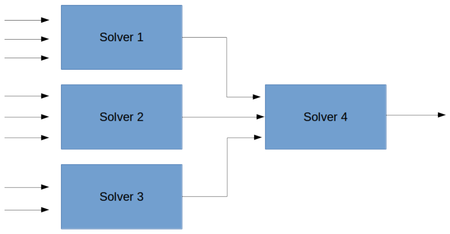
Abstract:

Since the beginning of space exploration, the number of orbiting space objects is dramatically increasing and critical Earth orbits, such as the Geostationary Orbit (GEO), are saturated with non functioning satellites. The "Loi relative aux Oprations Spatiales" (LOS, Law of Space Operation) legally obliges space companies like ArianeGroup to deorbit end-of-life objects and to ensure that the reentry in the Earth atmosphere of these objects presents no risk for human assets.

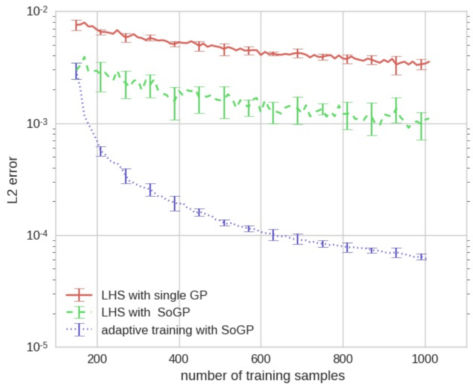
To assess the risk associated with a reentry event, ArianeGroup needs to predict the trajectory in the atmosphere of the reentering object, using a multiphysics modeling. At ArianeGroup, the trajectory of a reentering object is simulated using a system of solvers (SoS) consisting of a set of interdependent solvers coupled together. Specifically, the simulation involves a trajectory solver coupled with an aerodynamic solver, a fragmentation model, and an ablation solver. These physical models involve many unknown parameters and dedicated uncertainty quantification methods are needed to assess the reliability of the simulation-based predictions. Propagating uncertainties in a system of solvers can be challenging, due to the coupling effects on the dependences of the trajectory with respect to the uncertain input and the computational cost arising from the sequential evaluation of multiple solvers. In these situations, standard uncertainty propagation methods are too costly and alternative methods dedicated to SoS have to be derived. For instance, surrogate-based methods aiming at approximating the SoS as a whole may be extremely demanding, while exploiting the structure of the system can drastically reduce the computational effort [3].

In this work, we propose an original method for constructing a system of Gaussian Processes (SoGP) to form a surrogate model of a system of solvers. The SoGP is composed of a set of Gaussian Processes (GP) that reproduce the structure of the SoS under study. Each solver of the SoS is associated with a GP in the SoGP which is trained to approximate its corresponding solver. The prediction of the SoGP is not Gaussian as it is generally the composition of GP models [1]. This type of construction was developed in the case of 2 solvers in [2]. The advantages of the SoGP, compared to constructing a single GP for the whole system at once, are essentially the following. First, the SoGP has a richer structure and offer more flexibility, and therefore it can fit a larger range of functions. Second, training the SoGP requires learning multiple but usually simpler individual solvers, possibly adapting the training efforts. On the contrary, a global GP model needs to learn the (generally) more complex mapping between the SoS inputs and its outputs and requires the simulation of the whole system.

The important contribution of this work is the derivation of adaptive training strategies for SoGP. Adaptive learning is widely used to train single GP. For instance [4] proposed a learning algorithm based on the maximum of predictive variance (or Maximum of Mean Squared Prediction Error, MMSPE) to efficiently select new training samples in the input space. To reduce the SoGP prediction error more effectively, one wants to select distinct new training samples for each GP



(a) example of system of solver



(b) error convergence curve for three training strategies

Figure 1: The left hand side figure represents a system of solvers. The right hand side figure compares the evolution of the error function of the number of training samples for three different surrogate models. With *LHS with single GP* the whole system is emulated by a single GP and LHS training plan. With *LHS with SoGP* the system is emulated by several GPs assembled in a SoGP. The training is generated with LHS. With *adaptive training with SoGP*, the system is emulated by several GPs assembled in a SoGP and one of our adaptive training strategy is used.

constituting the system, and possibly to train only a selected subset of GP. To do so, we derive a predictive variance decomposition of the SoGP into contributions from individual GP. For practical use, unbiased estimators of these contributions are derived along with lower computational cost (but biased) approximations. The decomposition of the predictive variance is the backbone of training algorithms proposed subsequently, that identify the GP and its input point having the highest contribution to SoGP variance. The SoGP approach and the proposed training algorithms are tested on analytical problems. The tests show significant improvements compared to a single GP or a SoGP trained on simple LHS plans (see figures 1).
The SoGP framework is finally applied to construct a surrogate of the space objects reentry system of solvers used at ArianeGroup and predict the ground impact point. Using SoGP brings major improvements, in terms of precision, compared to using single GP model.

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Short biography – Francois Sanson is a PhD student in applied mathematics at Inria. He holds a master degree in aerospace engineering from Purdue University and engineering Diploma from ENSTA ParisTech.