Uncertainty Quantification for Stochastic Approximation Limits Using Chaos Expansion

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Abstract:

In a recent work [CFGS17] we propose a new efficient method for the uncertainty quantification of the limit of a stochastic approximation algorithm.

Typically, stochastic approximation (SA) is used to find zeros of a function that is only available in the form of an expectation, i.e. for solving equations of the form $\mathbb{E}[H(z,V)] = 0$, where V is some random source of noise. We study the uncertainty quantification (UQ for short) problem for the SA limits (the zeros), denoted by $\phi^\star$. The field of model uncertainty deals with the situation where the law of the random noise V is not known exactly. This is expressed in the form of a parametric dependence $V \sim \mu(\theta, dv)$ where the distribution of V depends on an unknown parameter θ for which only some probability distribution π is available. The uncertainty can also come from the function H, through a dependency in the uncertain parameter θ. Therefore, the equation to solve becomes

$$h(z,\theta) := \int_{\mathcal{V}} H(z,v,\theta)\mu(\theta,dv) = 0, \qquad \pi\text{-}a.e., \tag{1}$$

so that the zero $\phi^\star$ depends on θ, i.e. $\phi^\star = \phi^\star(\theta)$.

In this setup the UQ problem consists in determining the distribution of $\{\phi^\star(\theta) : \theta \sim \pi\}$. Among the possible methodologies we choose the chaos expansion, which aims at computing the coefficients of the function $\phi^\star$ on an orthogonal basis of the L^2 space with respect to the distribution π. The distribution of $\{\phi^\star(\theta) : \theta \sim \pi\}$ can then be approximated by an empirical distribution, i.e. by sampling independent and identically distributed θ and computing our approximation of $\phi^\star$ at the corresponding values of θ. Here, obviously, the most demanding part is the numerical computation of $\phi^\star$.

In [CFGS17] we introduce an iterative procedure (called the USA algorithm, Uncertainty for Stochastic Approximation) in increasing dimension for computing the chaos expansion coefficients of the SA limit $\phi^\star(\cdot)$ as a function of the unknown parameter θ on an orthogonal basis of the infinite dimensional L^2 Hilbert space. We design an SA algorithm for $\phi^\star(\cdot)$ so that each iteration lies in a finite dimensional subspace of the Hilbert space, while the dimension of these subspaces goes to infinity. Like usual SA algorithms, our algorithm is sequential, so that at any iteration it can be stopped and provides a numerical approximation of $\phi^\star(\cdot)$ with some controlled accuracy.

Beyond model uncertainty, applications of our approach include sensitivity analysis, with respect to θ, or quasi-regression in the sense of reconstructing a whole unknown function, for instance in the context of nested Monte Carlo computations involving a nonlinear inner function $\phi^\star = \phi^\star(\theta)$.

In [CFGS17] we prove the a.s. and the L^p $(p < 2)$ convergence of the USA algorithm to the true function $\phi^\star(\cdot)$ with respect to the Hilbert space norm. We provide explicit sufficient conditions for the convergence in terms of finite dimensional problems for fixed values of θ, as opposed to abstract assumptions involving Hilbert space notions in the previous literature on the subject.

Our algorithm is fully constructive, detailed, and easy to implement. The theoretical convergence result is supported by numerical evidence. We also analyze numerically the behavior of the algorithm with respect to various design parameters and provide extensive reports and discussion on numerical tests.

In a subsequent work (which is in progress) we analyze under additional assumptions the L^2-convergence rate of the USA algorithm that allows the non-asymptotic error control.

References

[CFGS17] S. Crepey, G. Fort, E. Gobet, and U. Stazhynski. Uncertainty Quantification for Stochastic Approximation Limits Using Chaos Expansion. 2017. Submitted preprint, available at `https://hal.archives-ouvertes.fr/hal-01629952`.

Short biography – I hold the diploma of Ecole Normale Superieure (Ulm) in Mathematics, 2013-2016; Master degree in Probability and Statistics of Université Paris Sud, 2014-2015; Master degree in Financial Mathematics of Ecole Polytechnique - UPMC (ex-DEA El Karoui), 2015-2016. I also passed a 6 month internship as a quant researcher at Société Générale (Apr. 2016 - Oct. 2016). Currently I'm on the 2-nd year of PhD with prof. Emmanuel Gobet. I have a 3 year PhD funding from ENS (Ulm). The main research directions of my thesis are Uncertainty Quantification, Discretization and Inference for Stochastic Processes.