Quantile prediction of a random field extending the gaussian setting

A. Usseglio-Carleve
Université Lyon 1

Supervisor(s): Prof. Maume-Deschamps (Université Lyon 1) and Dr. Rullière (Université Lyon 1)

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Adress: Institut Camille Jordan
43, boulevard du 11 novembre 1918
69622 Villeurbanne cedex

Email: usseglio@math.univ-lyon1.fr

Abstract:

Let us consider a random field $(Z_t)_{t\in T}$ observed at N points $t_1, ..., t_N \in T$, where typically $T \subset R^d$. A widespread problem is to predict the value Z_t of the field at point $t \in T$ given $Z_{t_1}, ..., Z_{t_N}$. For that purpose, kriging [3] consists in taking the conditional mean of Z_t given $Z_{t_1}, ..., Z_{t_N}$. In a gaussian setting, explicit formula are straightforward to obtain. Furthermore, this approach may be generalized by taking the $\alpha-$quantile of $Z_t|Z_{t_1}, ..., Z_{t_N}$, where $\alpha \in]0,1[$. Indeed, for symmetric distributions, the mean and the $0.5-$quantile coincide, then kriging may be seen as the particular case $\alpha = 0.5$. On the other hand, the cases $\alpha < 0.5$ and $\alpha > 0.5$ are interesting to provide confidence intervals.
Finally, the problem introduced above consists in estimating the quantile of a component of a vector conditioned by other components. Obviously, if the random field, and therefore the vector is gaussian, then conditional distribution is still gaussian, and explicit formula are known. In this work, the aim is to extend the gaussian assumption to elliptical distributions [1]. Leaving the gaussian framework, there is no longer stability by conditioning, this is why we cannot get explicit formula for conditional quantiles.
A first approach may be to approximate the $\alpha-$quantile of Z_t given $Z_{t_1}, ..., Z_{t_N}$ by an affine combination of $Z_{t_1}, ..., Z_{t_N}$. This approach, called quantile regression [2], is used in the gaussian setting and performs well. We thus provide explicit formula for quantile regression predictor in the elliptical case, and show through numerical examples that such a predictor performs very poorly leaving the gaussian case, especially when the quantile level α becomes close to 1 [4].
To avoid this problem, we propose another approach, by etablishing an asymptotic relationship between unconditional and conditional quantiles, when α goes to 1 [4]. Such a relationship, based on two parameters η and ℓ, allow us to estimate more easily the conditional quantile through the unconditional one (see Figure 1).
Finally, in the case of heavy-tailed distributions, we propose some estimators for parameters η, ℓ, and therefore for conditional $\alpha-$quantile. We provide consistency and asymptotic normality results, and illustrate that with a simulation study, and a real data example [7].
To conclude, we propose a similar approach, considering expectile (another risk measure introduced in [6]) instead of quantile [5].

References

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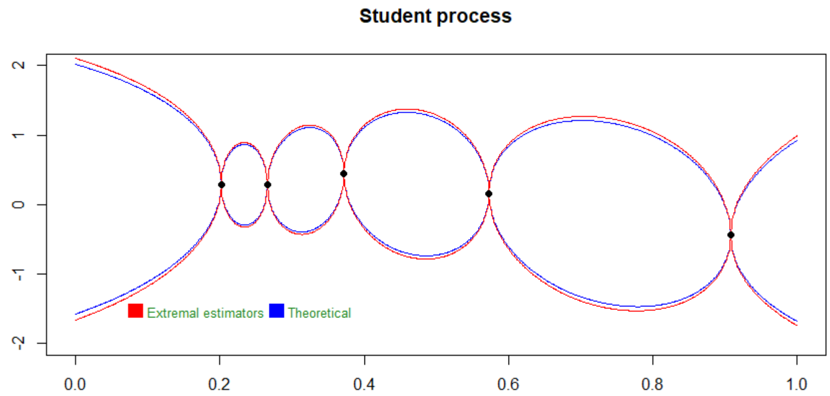

Figure 1: Quantile approximation from the unconditional distribution for a Student process observed in 5 points, and a quantile level $\alpha = 0.999$.

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Short biography – I obtained a Master of Research in Actuarial Science in 2015. My Master's thesis on black-box optimization for Economic Scenario Generators led me to conditioned quantiles estimation in my PhD thesis, funded by Université Lyon 1. I am also currently working on an application of this work for temperature random fields, and financial data.