Surrogate-Aided Adaptive Multi-Fidelity for Low-Cost Optimization Under Uncertainty with the SABBa Framework

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Mathematical problem

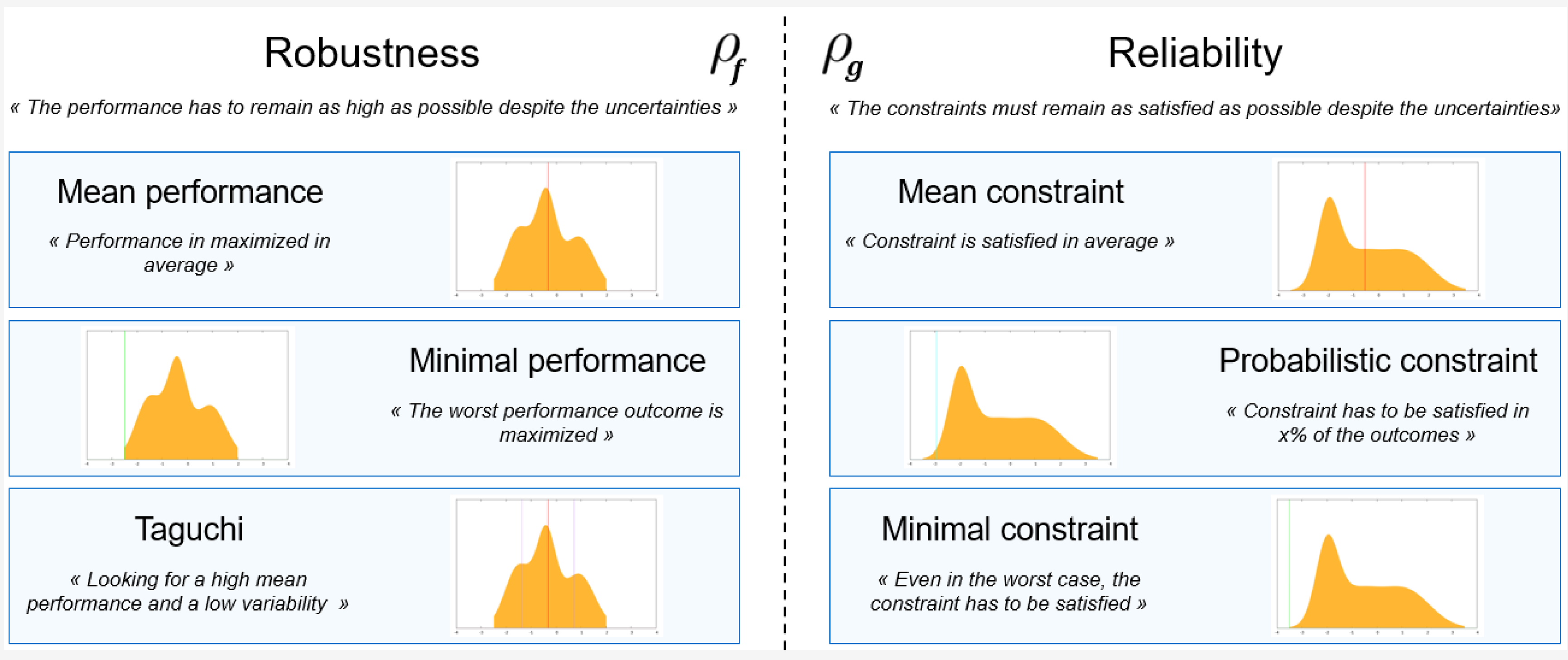
- Find a robust or reliable optimum by replacing the objective and constraint functions with robustness and reliability measures.

$$\begin{aligned} \text{minimize/maximize:} \quad & \boldsymbol{\rho_f}(\boldsymbol{x}) \\ \text{subject to:} \quad & \boldsymbol{\rho_g}(\boldsymbol{x}) \geq \mathbf{0} \\ \text{by changing:} \quad & \boldsymbol{x} \in \mathcal{X} \end{aligned}$$

- Many choices for $\boldsymbol{\rho_f}$ measures (same for $\boldsymbol{\rho_g}$):

$$\begin{aligned} \text{Expectation} \quad & \boldsymbol{\rho_f}(\boldsymbol{x}) = \mathbb{E}_{\boldsymbol{\xi}}\left[\boldsymbol{f}(\boldsymbol{x},\boldsymbol{\xi})\right] \\ \text{Variance} \quad & \boldsymbol{\rho_f}(\boldsymbol{x}) = \mathrm{Var}_{\boldsymbol{\xi}}\left[\boldsymbol{f}(\boldsymbol{x},\boldsymbol{\xi})\right] \\ \text{Min/Max} \quad & \boldsymbol{\rho_f}(\boldsymbol{x}) = \min_{\boldsymbol{\xi}}\left[\boldsymbol{f}(\boldsymbol{x},\boldsymbol{\xi})\right] \text{ or } \max_{\boldsymbol{\xi}}\left[\boldsymbol{f}(\boldsymbol{x},\boldsymbol{\xi})\right] \\ \text{Quantile} \quad & \boldsymbol{\rho_f}(\boldsymbol{x}) = \mathrm{q}^p_{\boldsymbol{\xi}}\left[\boldsymbol{f}(\boldsymbol{x},\boldsymbol{\xi})\right],\ p \in [0,1] \end{aligned}$$

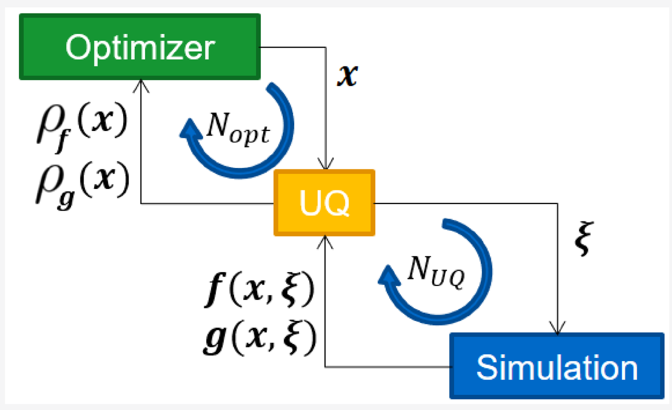
which allows several paradigms:



<u>Main issues</u>: Curse of dimensionality, limited computational budget for measure computations

Classical methods

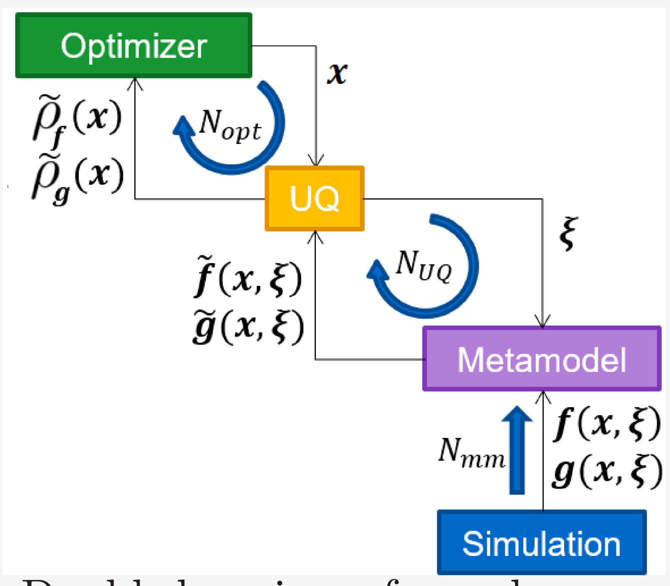
Double loop / Nested loop



At each optimization iteration, a full uncertainty quantification is performed and the measures are calculated for the current design $\boldsymbol{x}$.

Issues: High cost, no memory

A Priori MetaModel



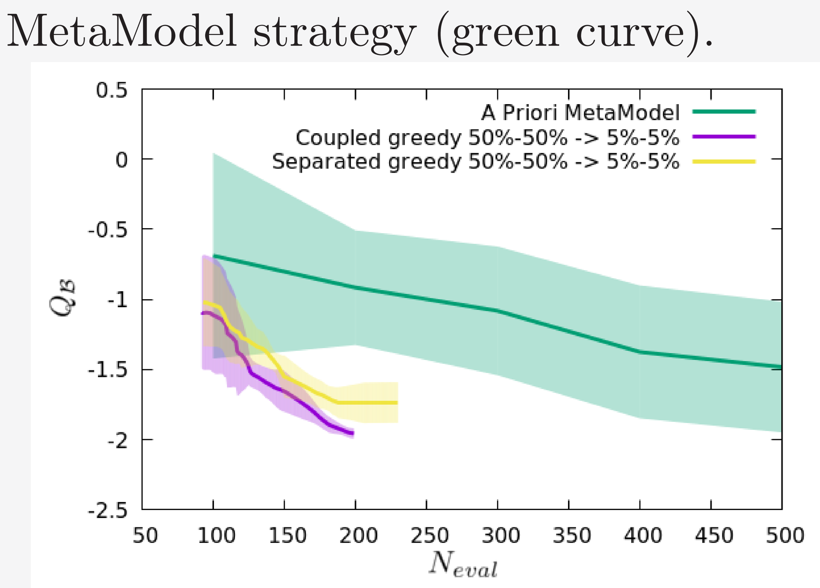
The Double loop is performed on a metamodel (negligible evaluation cost) built in the coupled space $(\boldsymbol{x},\boldsymbol{\xi})$ over a chosen design of experiments.

Issues: Choice of N_{mm}, no adaptivity

Convergence analysis

- SABBa is performed with coupled-space (purple curve) and separated-space (yellow curves) models. The problem is a bi-objective **mean/variance minimization**.
- Ten runs of each approach give mean convergence and associated variability. Worst runs are printed on the right to assess the robustness of the framework $\Longrightarrow$
- The log-scaled probabilistic modified Hausdorff distance to the optimum is plotted.
- Improvement with respect to an A Priori MetaModel strategy (green curve).

with high-quality metamodel



with low-quality metamodel

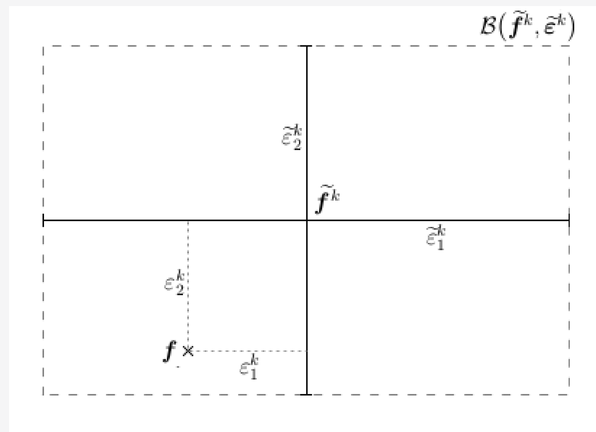
Perspectives

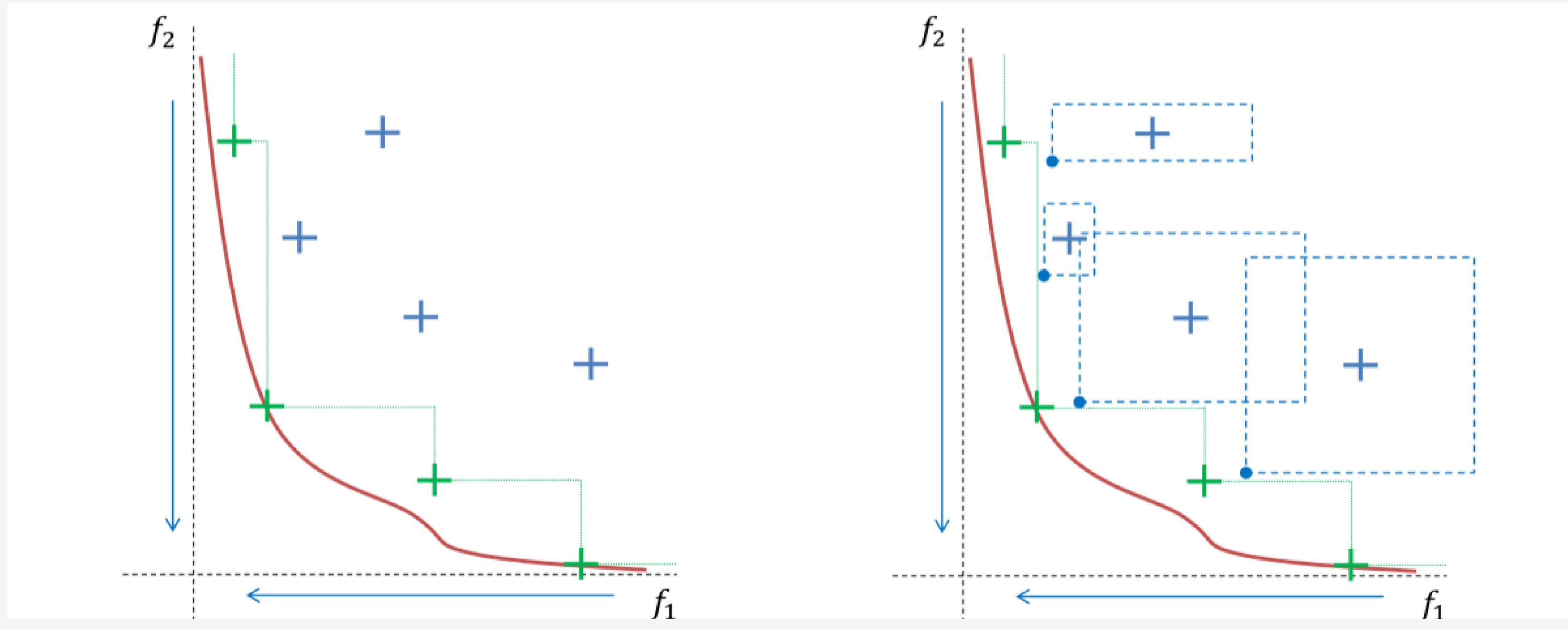
- Complete the analysis by considering additional problems/measures.
- Apply the framework to several engineering-based optimization problems.
- Extension to bayesian optimization under uncertainty.

SABBa framework

1- Bounding-Box approach

- Box-approximation of the statistical measures, containing the real value.
- Uncertainty Quantification is performed accurately only on promising designs, creating an adaptive UQ multi-fidelity.

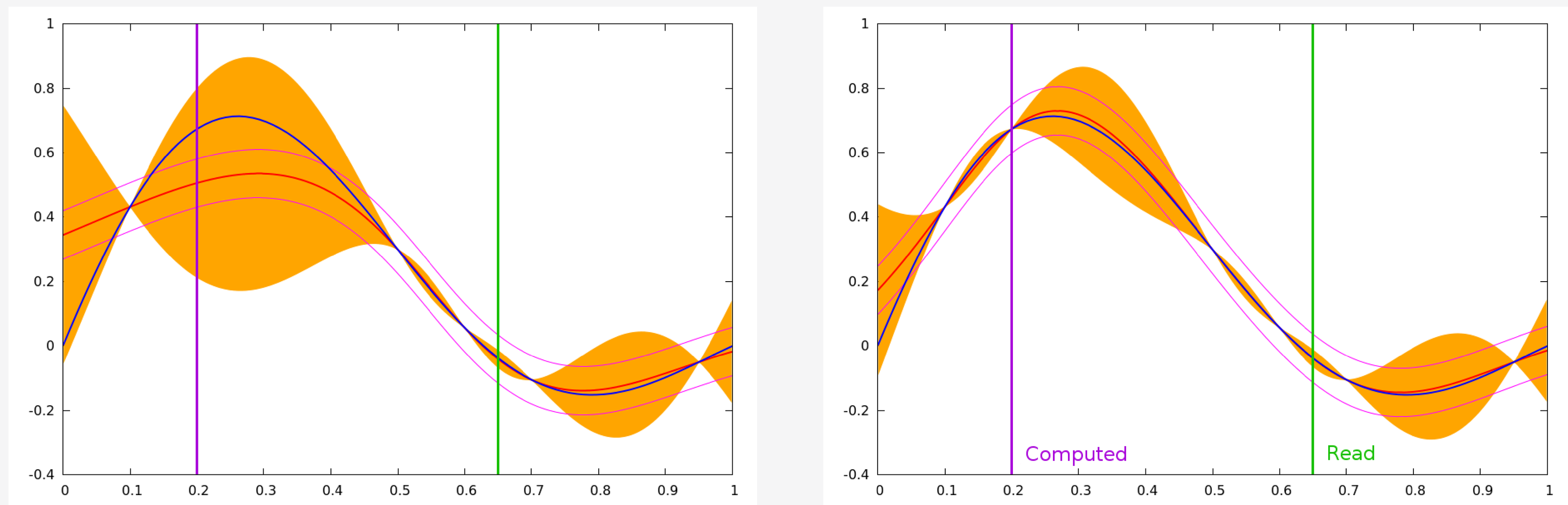




→ **Cost-reduction for the computations on non-optimal designs, at the beginning of the optimization**

2- Surrogate-Assisting strategy

- Metamodels are built on the statistical measures and updated at each evaluation to bypass the UQ process when convergence is reached.



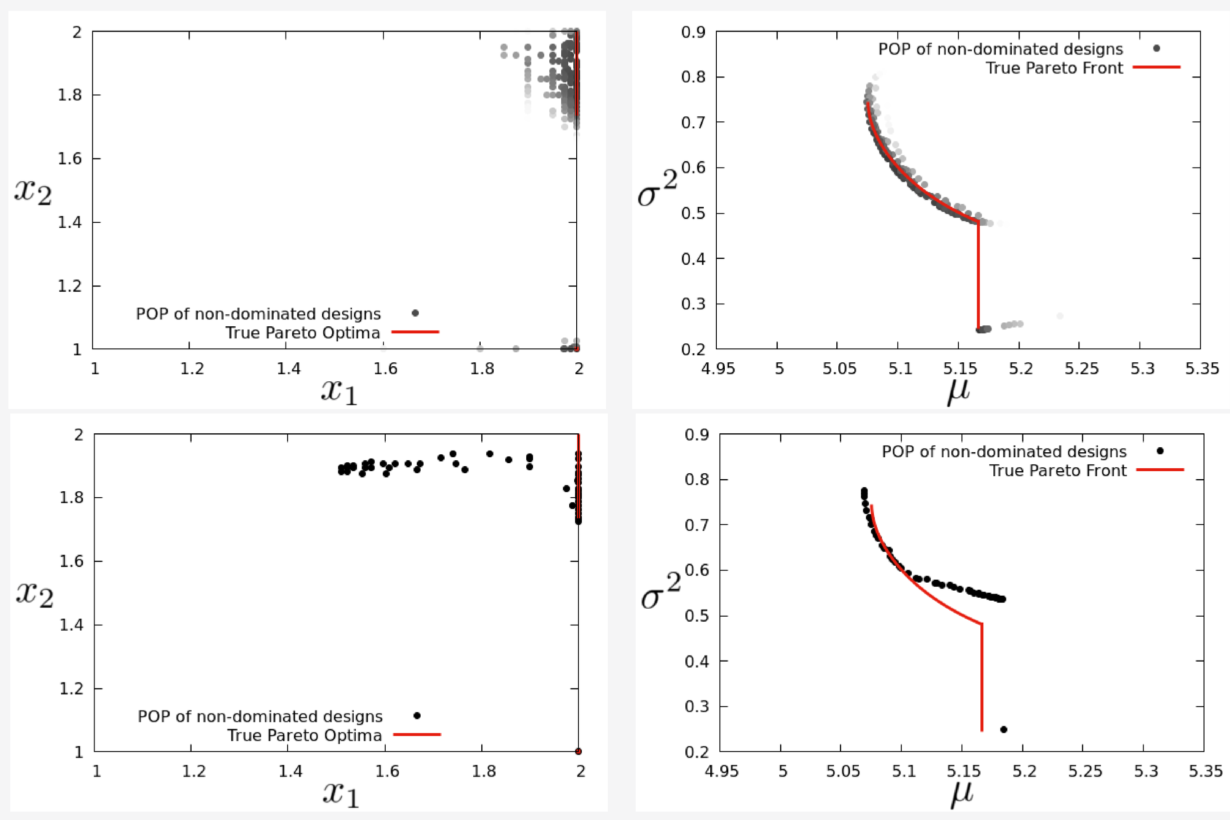
→ **Cost-reduction of the densification of the Pareto front, at the end of the optimization**

Results (worst from convergence analysis)

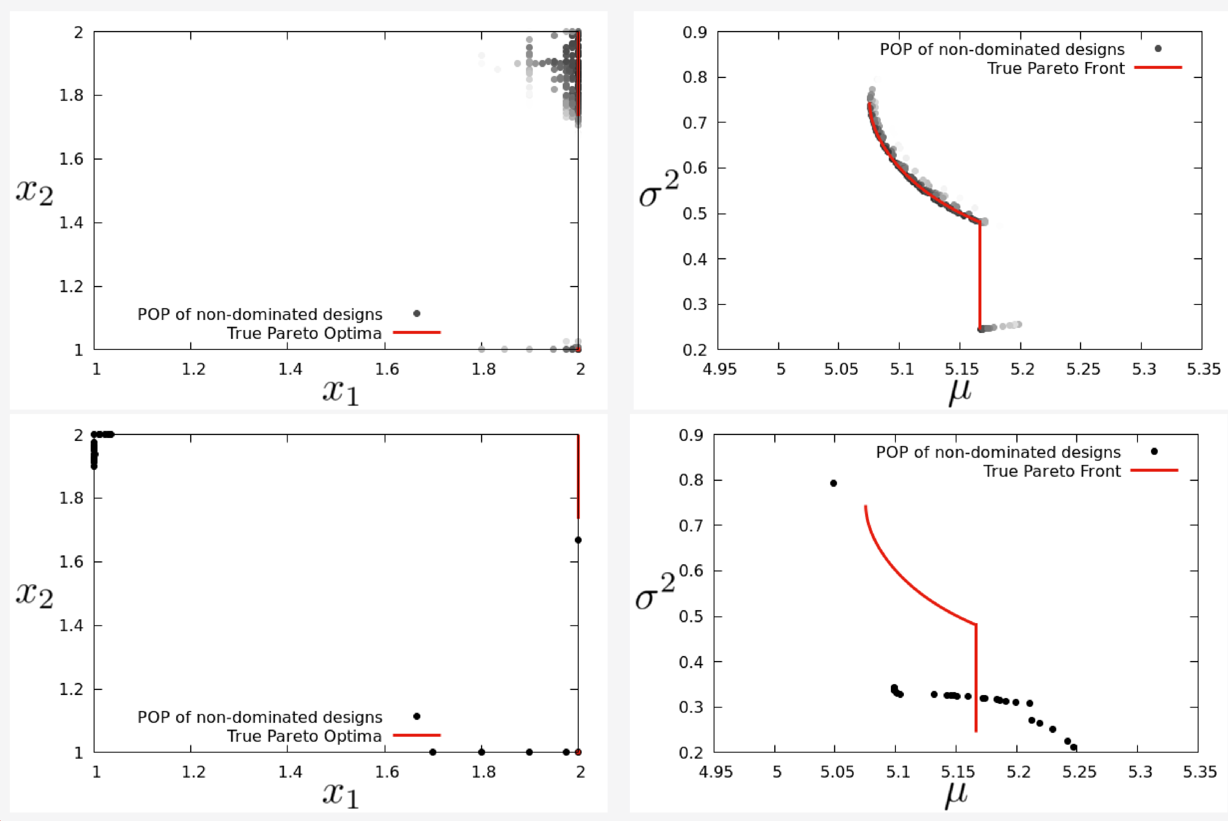
High-quality metamodel

- SABBa with coupled-space metamodel for statistics computation.
95 evaluations

- A Priori MetaModel strategy.
100 evaluations



Low-quality metamodel



- SABBa with coupled space metamodel for statistics computation.
196 evaluations

- A Priori MetaModel strategy.
200 evaluations

References

M. Rivier, P.M. Congedo. Surrogate-Assisted Bounding-Box Approach for Optimization Problems with Approximated Objectives. *[Research Report] RR-9155, Inria. 2018, pp.1-35.*

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